THE MATHEMATICS TEACHER

Volume XXXV Number 5

Edited by William David Reeve

Mathematics in Our Schools and Its Contribution to War*

By Sophia H. Levy

University of California, Berkeley, California

Less and less mathematics has been taught in our schools during recent years; fewer courses have been given than formerly, and subject matter in courses remaining has been diluted until at times the entire character of the material presented has been changed. Meanwhile we have heard that courses in mathematics must give way to courses in social studies of one kind or another. The outlook for mathematics has been growing darker and darker during recent years.

It has at times been most difficult to maintain an optimistic attitude concerning the fate of mathematics in our secondary schools: general mathematics replacing algebra as a ninth-grade subject, even for the pre-engineering, mathematics, and science students; trigonometry often being advanced from the high school to the college level; to say nothing about the postponement of the topics in arithmetic. You teachers of mathematics are all too aware of the situation to which I am referring. I have watched our classes in engineering mathematics become less and less able to handle even the simpler parts of arithmetic and algebra, which should have been part of their normal equipment. But, they were no less capable than students of earlier years.

During the last year some have expected that school administrators and educators would find themselves forced to change their views concerning the importance of mathematics in the secondary program because the need for the subject became increasingly evident as we proceeded in our program for defense. And now a certain, and sometimes considerable, knowledge of mathematics is demanded in so many branches of our armed services, as well as in defense industries, that school administrators and the public are suddenly more friendly towards mathematics and are now willing to see it taught in the high school—taught even to the extent of having it occupy its rightful place of prominence in the curriculum. Neither private school headmasters nor public school officers could stand up under the pressure of having hundreds of their candidates for commissions rejected by the Navy because of deficiencies in mathematics. The youths who really know their mathematics can usually apply it reasonably well, whether for defense or for other ends, and the need of their services is recognized.

* Read at the Annual Meeting of The National Council of Teachers of Mathematics at San Francisco, February 21, 1942.

Parts of my paper will summarize replies which I have received in response to a letter I addressed to several hundred teachers and educators asking about work in mathematics for defense offered in their communities. More than one hundred replies have been received and they indicate that much progress has already been made. I feel that this is due to the wisdom of the teachers of mathematics, for they had not lost faith in their subject and were ready for the emergency.

One teacher writes: "We have always felt as though we were an isolated community, since we are forty-four miles from the nearest town and have no bus route or railroad. But since war was declared, we have an airport and bombing range, and the cavalry is stationed here to guard the mine and the border. We are designated as a war zone." This teacher now has five regular classes in mathematics, tutors boys preparing for service who need the subject, and who cannot get it because of schedule conflicts, is reviewing algebra for several young men who now must continue on in mathematics, and next year she is to give trigonometry even though her state university does not accept a high school course in that subject. And—in order that her time might be sufficiently occupied, apparently—if the soldiers decide to take courses in mathematics, she is to hold evening classes for them. This teacher is a woman who had neither major nor minor subject in college in mathematics. She built up her preparation after her first assignment, which was to teach beginning algebra, through summer session work and through one year at a university. She and the type she represents are shouldering their responsibilities in this war.

Many of the letters which I have received have been from sizable schools and school systems. It is my hope that smaller schools are also becoming aware of the importance of the study of mathematics in our defense and war efforts and are carrying forward their programs in mathematics along similar lines.

Many schools report a very large increase in the number of students in mathematics courses, both in the academic branches and in the shop classes. As teachers, we all welcome the opportunity to present our subject to these increasing numbers.

Many schools report that this increase has been voluntary, while others indicate that wise counselors are impressing upon the student his own need of mathematics and the need of his country, and that it is his patriotic duty to become as well prepared as possible to meet such demands as may be placed upon him.

The list of shop and trade courses which require some mathematics is a formidable one: sheet metal trades, ship fitting and other ship-building crafts, tool making, marine pipe fitting, surveying for artillery, aircraft work, drafting, navigation, the making of gears, reading of blueprints, radio, pattern making, machine tool design, meteorology, and nursing.

We cannot tell how future pressure on us in the matter of defense will affect the teaching of mathematics. You may have a group of students, for example, who need trigonometry badly. Their previous training may not have been recent. I think it behooves us to see that a proper review of the essentials of arithmetic and algebra is secured.

We may be confronted with an urgent need for preparing students in certain fields of mathematics in relatively short periods of time. I sincerely trust that some way to meet this situation may be worked out with due regard for the fact that one cannot cut down very drastically on the standard length of time being devoted to mathematics courses in our secondary schools. Longer class periods or an increase in the number of meeting periods might solve this problem. Better still, though, is the plan to prepare for national defense through enrollment in the

fundamental mathematics courses before further emergency arises.

We should all be grateful that we have found in our schools the fund of mathematics ready for those students and those men in defense trades who find they need it so urgently at this time—grateful to the teachers who have continued to do the best they could to present mathematics during these recent years, in which principals, superintendents, and curriculum builders have tried so hard to block, undermine, and discourage their efforts. It is true that in some localities the character of the school population has changed very markedly during recent years, and schools have had to care for pupils of lesser general preparation and lesser abilities. Many problems have arisen. True it is that the education of these pupils for good citizenship and lives useful to themselves, their communities, and their country is one of the most important functions of our schools now and forever, but we must oppose and continue to oppose the efforts in many localities, in many parts of our country, to offer but a single program of mathematics and that fitted to the needs and capabilities of the weaker student. More than ever we must now protect the training of the good student.

I wish I might state to you that all schools now placing additional stress on the subject of mathematics were doing so in a sound manner. My findings are that some educational systems think they are meeting the demands of the present emergency by offering courses which they describe as extremely flexible and effective and practical, aimed to equip the student with a working knowledge of mathematics, which will shorten the training time required when he enters either the armed forces or industry. Now these words sound good, but when you find that this means that the course consists in the perusal of sets of formulas in handbooks, supplemented by comment from the teacher on how to substitute in the formula to get the answer desired, you will agree that we should all be on our guard so that this cannot be the type of course we are asked to add to our schedules as our patriotic duty in the name of defense.

One system states that by emphasizing the applications of mathematics and not the derivation of formulas they are able to give in one year what was formerly given in two and one-half years, thus releasing time for the brighter students to use in obtaining other skills. Such comments are vacuous.

On the other hand, I am grateful I can report that the majority of schools from which I have heard are emphasizing the need for more thorough foundation work than they have been accustomed to in the immediate past. I trust that this indicates that we are returning to our schools the best parts of our older forms of instruction.

Secondary teachers have in recent years given up, or have been forced to give up, what I consider one of their privileges—the teaching of trigonometry. Why administrators and curriculum builders should decide that trigonometry is a subject which belongs on the college level, I cannot say. True it is that colleges should provide instruction in this subject, for there will always be students from smaller school systems who have been unable to secure trigonometry in the secondary schools, due to bona fide administrative difficulties. But the subject of trigonometry should remain in our secondary schools. The information which I have been receiving all points to an increasing demand for it, taught either in the high school or in college. I have been informed that some secondary schools have recently added trigonometry to their programs, to meet demands of aircraft production companies, other defense industries, and government agencies now wishing more men available with a knowledge of this subject. I trust that when the emergency is over, trigonometry will have found a permanent place in our secondary system.

We must not forget, either, that mathematics, including trigonometry, will be needed during the reconstruction period which must follow the war.

I have always been deeply concerned over the fate of arithmetic. No matter what the grade of the teacher you speak to, you find it the common experience that students cannot handle problems in this subject. Yet curriculum planners seem to think that less and less arithmetic need be taught. We have a modern version of the 3 R's—*Refresher* arithmetic, *Remedial* arithmetic, and *Review* arithmetic—the companions of the weaker parts of progressive education. Why must arithmetic be constantly "remedied?" It certainly is given in very small doses and sugar-coated, so that the taste can hardly be recognized! Maybe the demands of the war will bring about *Revival* arithmetic.

A letter by C. W. Nimitz, then Chief of the Bureau of Navigation, now Commander of the Pacific Fleet, containing observations on an examination given to 4,200 entering freshmen at twenty-seven of our leading universities and colleges, shows that a shockingly high percentage were unable to pass an arithmetical reasoning test. In order for one school to enroll the necessary number of men in the training course, it was found necessary to lower the standards in fifty per cent of the admissions, this necessity being attributed to a deficiency in the early educations of the men involved. A study was made of the grades received in examinations by candidates for admission to the Navy, the grades being classified according to the location of the recruiting station through which the candidate applied for enlistment. The proficiency in arithmetic in the eastern part of the country was strikingly greater than that of the middle west and west. The *lowest* average mark *east* of the Mississippi was equal to the *highest* average mark *west* of the Mississippi. The three highest average attainments in arithmetic were all in New York state.

Professor C. N. Schuster, Chairman of the Committee on Defense of the National Council of Teachers of Mathematics, has talked with a large number of technical experts in various fields of defense and applied mathematics, and I would like to present to you a few of the remarks which he has made to me. It was the opinion of the majority of these experts that we should teach all the mathematics we can possibly get into our programs. It should be of the type that has applications, but in the main the technical applications should be left to the experts in the various fields. The teachers of mathematics should be stressing the fundamentals of their subjects, and on this solid foundation the experts can build the necessary proficiencies required in the defense fields. When applications are taught, the purpose should be to create interest, rather than to prepare the student for work in specific defense fields. Applications should be of the type which do not involve technical terms nor require instruments that cannot be obtained in the schools.

Granting that to meet war requirements, our need is not for new courses, but rather for the strengthening wherever possible of the teaching of the fundamentals of our subject, I might add a few general recommendations concerning points at which teachers may well place some extra emphasis at this time.

Stress the need for *accuracy* in mathematics. Have students realize the difference between *absolute accuracy* and *sufficient accuracy*. There should be no general rule such as is found in some textbooks, that calculations be made to this or that number of places. There should be understanding of the term, *significant digits*. Accuracy, meaning absolute accuracy, should be appreciated. Checks as a means of discovering inaccuracies, in order that they may be eliminated, should be emphasized. Under this same heading of accuracy, include *integrity*. Those "educators" (not teachers) who are needlessly fearful of developing complexes in students, will not agree that ignorance should

be confessed; but how else is one to improve? Everyone in this room has certainly had many experiences with pupils who try to cover up the fact that they do not know the answer to a question. Where correctness of result may mean the saving of lives, integrity becomes important. It goes without saying that the teacher's own attitude toward his or her own limitations of knowledge should be an example to the student. Both the student and the teacher should realize that confession of ignorance becomes increasingly important where human lives are at stake.

Emphasis should also be placed on *caution*. A student should be kept from making snap judgments. Caution is very different from taking a chance on any old answer in the hope that it may be the correct one. The student who indulges in snap judgment may be endowed with overconfidence or perhaps only laziness. A simple example of a case where caution must be exercised is illustrated in the following problem: A circle rolls on the outside of another of twice its radius, and goes completely around the second circle. How many times does it revolve? Verification is simple, but without caution a wrong answer may easily be given. A hasty answer is worse than none.

The element of competition among pupils should be suppressed, for it is important not to discourage the slower pupil, who may be, in reality, the better one.

The student should be cautioned against attempting to answer any question given in too general wording. The wording must be precise. For example: A man walks around a tree. Does the tree go around the man? Unless the expression "go around" is given a meaning, the question cannot be answered.

See that the *rule of signs* in generalized subtraction is understood. See that students understand that -4 is less than 1. Proportions, fractions, and percentages need to be stressed. You probably cannot imagine a university student simplifying 16/64ths by drawing a line through the 6's. What is the probability that one will get the correct result from such procedure, as this student did? I have recently battled with a man, without success, trying to show him that adding $\frac{1}{2}$ and $\frac{1}{2}$ gives two halves. He insists that he will get $\frac{2}{4}$. Many of you probably feel, as I say this, as a friend of mine did when I told her about it. She said: "When you find the solution, I'll be glad to have it. I have the same problem."

I think that most of the university students who have no better comprehension of arithmetic than the two I am quoting were building dollhouses and Indian teepees during the school time in which arithmetic should have been studied.

More time should be given to the study of irrational numbers, including square roots. The latter subject, I understand, is now on the list of outcasts. The curriculum builder who postpones long division to the sixth grade, would, I admit, if he attempted to teach square roots, have difficulty with students whose arithmetic program had been as dilute as many in our schools.

Considerable practice in the use of logarithms should be given, and, where possible, approximate computations using the slide rule.

We all know the difficulties our students have with word problems. A great amount of this difficulty is literary. But as every real situation presenting a need for mathematics is presented in words, I see no way in which we can get along without word problems, modern theories to the contrary notwithstanding. The student must be able to understand quickly and fully the written and the spoken word; and, equally important, he must be able to put his own thoughts in written or spoken language comprehensible to others. Language is the complement of reason.

If you are pressed for time, do not hesitate to enlarge on the assumptions you are making, but be careful that your deductions are consistent with the assumptions made, and that the students understand

the relationship between assumptions and conclusions. An example of what I mean may be taken from the study of logarithms. In our war effort the use of logarithms is exceedingly important and plenty of practice must be given. Skills once acquired must be retained through practice. I see no harm, and I can see much good, resulting from spending extra time on the use of tables, putting to that use time which might ordinarily be spent in discussion relative to the construction of the tables.

In problems on similarity, including similar areas, make use of graph paper on which the squares are of different sizes. Plot two figures of the same dimensions on papers of different rulings and you can read from the two figures equal answers for equal parts, whereas if you measure lengths of corresponding lines of your two figures using but one scale of measure, any ruler for example, corresponding lengths will be in proportion, corresponding areas will be in the ratio of the squares of corresponding lengths, and so on. Your own experience will suggest countless uses of this method.

We should place the emphasis in mathematics courses on quality of work, rather than on quantity. It should be borne in mind that the preparation is for defense industry as well as for service in the armed forces, not forgetting meanwhile the preparation of our students for return to civil life. We should discontinue the common practice of allowing the inadequately trained to "pass." Certainly the man inadequately prepared should not obtain a position of responsibility where his decisions affect the safety of the men with whom or over whom he works. He is an ineffective member of an organization and is a source of danger to the whole, rather than just so much waste. Without being unduly pessimistic, it may safely be said that there is evidence of weakness in the kind of training offered in mathematics in preparation for war, for example in arithmetic, and this will not be without effect on the military effort. When we hear that 100,000 men signed up in the Navy during the months of December and January alone, we have a suggestion of the amount of skill and competence required to make this large force useful.

We ask the question as to the amount of mathematics desired by our Army and Navy. Peacetime preparation is indicated by the amounts required for graduation from West Point and Annapolis, whose high requirements are known to all of us. Wartime preparation, especially preparation which must be secured in the short time at our disposal, is a different matter. Teachers of mathematics are, I hope, familiar with the work of the sub-committee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America, which was published with the cooperation of the National Council of Teachers of Mathematics and the Central Association of Science and Mathematics Teachers. This important article stresses the need of mathematics in our secondary schools and points out the many places in which preparation will further the defense effort. I commend it to all of you for serious study. I might mention in passing the need for practice in the less familiar angular measure known as the mil, which is defined as 1/6400 of a circumference. Scales used by the artillery are commonly graduated in this unit.

I have called at the office of Naval Officer Procurement of the Twelfth Naval District, asking about the question of preparation and the need for preparation in mathematics for service in the Navy. I have been assured that I could not advise you people too strongly to encourage your students to take as much mathematics as you can possibly give them in the secondary schools, to be followed for those who come to the university by as much mathematics as we can possibly give them. *The Navy needs trained men.* It is, again, not a question of quantity, but rather a question of quality. Often the first questions asked

in interviews are: What mathematics have you had? What grades did you make in mathematics?" And the problem of the authorities is to see that those men with preparation in mathematics are put where they use mathematics, for the significant reason that the knowledge of mathematics is needed, and needed urgently, for the successful pursuit of this war.

I would like to bring to your attention a few of the new types of training which have been set up by our Navy to meet the present emergency.

For the procurement of prospective candidates for appointment in the U. S. Naval Reserve, a classification designed as V-7 has been established. The educational qualifications for these midshipmen include the possession of one of certain bachelor's degrees from an accredited university or college, which must include at least two one-semester courses in mathematics of college grade, including plane trigonometry. During the current semester at the University of California in Berkeley we have two classes of six units each in which we are trying to give trigonometry and the algebra essential to it, to young men who have been accepted in the V-7 classification by our Navy Department. Most of them are graduating seniors majoring in English, Journalism, Political Science, Economics, and History, who took one year of algebra and one year of geometry some time ago. We have three sections of a new course, Spherical Trigonometry and Navigation; another new course, Exterior Ballistics; and many new sections of algebra and trigonometry.

There are other classifications, V-5 for example, for appointment in the U. S. Naval Reserve, which also require college work in mathematics. The men preparing for aviation in the U. S. Naval Reserve, the V-5's, must have considerable work in mathematics, and they are given a syllabus through which they prepare for an examination which must be passed before their training actually begins. Here again is stressed the importance of learning and understanding the fundamental principles of mathematics; and for these men, also, a need for combination of speed and accuracy in the solution of problems. It is pointed out in an official circular of information that neither speed nor accuracy is possible without the knowledge of fundamental principles and some degree of order and style in their use; and to these people the knowledge of fundamentals is utterly useless unless problems can be solved quickly and with absolute accuracy. The naval pilot who flies alone at night over a foggy sea must be able to find his way to his objective and back to his carrier without the aid of radio beams, land marks, or air beacons. Should he make a simple mistake in addition or subtraction, in the use of fractions, or the placing of a decimal point, the penalty may be failure to carry out his mission and perhaps the loss of his life. The circular goes on to say that there is no sympathetic teacher to show the naval pilot his mistake, no congenial mechanism on the plane which refuses to follow a fatal course, no warning that the arithmetic is faulty, until the crash comes. And then, it's too late. These are harsh and serious words, but true; and there is no place in the naval service for the aviator whose knowledge and accuracy in dealing with mathematical processes is not sufficiently precise to insure his own safety and the safety of his men and equipment. The student is urged, therefore, to view mathematics in a new light . . . as an essential and fundamental tool in the practice of naval aviation without which men, planes, and perhaps the safety of the country are forfeit.

I have not been able to speak directly with a personnel officer of the Army, but one of our young people, an officer in the U. S. Army Reserve, did so for me, and though, under these circumstances, I cannot make a direct quotation, I can assure you that the Army, as well as the Navy, demands considerable use of mathematics in many of its divisions of service.

Let me quote from the September, 1941,

issue of the *Monthly Science News*, an English publication:

> This is a highly technical war. Much of it is a war of physicists and of light electrical engineers. Only countries with a large number of men trained in these subjects can really fight a modern war; only Great Britain, the United States of America, Germany and Russia are adequately equipped in this specialized man-power; and even they find it stretched to its limit. It has thus become an imperative need in the war, first, to see that all existing scientists are used to the best of their capacities, and second, to train young men in the key sciences . . . .

A register of trained people was made, and, to continue the quotation:

> The total number now registered in physics, engineering and chemistry is in the hundred thousands, and each of these names has been studied to see that the man is playing his fullest part in the scientific war. Many hundreds have been appointed to the government research laboratories; many hundreds to the ordnance factories; thousands to technical commissions, in all three services; thousands are doing essential research, development, and production work in industry; and thousands have to be left in the universities and schools to perform the equally essential task of keeping up the supply of scientific men.

Need a teacher of mathematics wonder whether or not he or she can contribute to the war effort?

Why Mathematics

By Richard Record*
Central High School, Oklahoma City, Oklahoma

The advantages of studying mathematics are many, and are so numerous and in some cases so subtle that one can scarcely recognize them. Mathematics is an exact science, an art, the beauty and value of which cannot be reckoned in terms of dollars and cents or any other exact standard by which we judge values today.

The average individual first becomes acquainted with the science of mathematics when he is in the elementary grades—the subject at once becoming distasteful to him perhaps because of prejudices handed down by older pupils or possibly because little Johnny wants to get out and play on the see-saw, but "6 plus 8" has him stumped. Consequently, by the time Jane or John reaches high school this inert fear of the numerical, has begun to manifest itself in the form of: "But Mother, I can't see why on earth I have to take this old algebra. I don't see what good it will ever do me anyway." Well, the foregoing may be an example of why the average student is thinking; but more and more, every day in the newspapers, in magazines, from biographies and from just plain verbal language everyone is becoming more "mathematics conscious."

Mathematics is the cradle of science, which has given us such modern conveniences as the automobile, the airplane, the incandescent lamp, the icebox, and countless other inventions and appliances which would have thoroughly startled C. Columbus. Mathematics may be called the veritable "backbone" of our academic standards because it is the basis of all system. The very books that we read are classified according to mathematical logic—the page numbers. The alphabetical index is in a sense mathematical—chronological. The calendar is mathematical, the clock is a mathematical instrument, chronometer, without which "clock-watchers" wouldn't exist. The desk is a geometric solid, the volume, surface area, weight, all of which may be calculated by mathematical formula. The symphonies, concerts, swing sessions are all music and have their very existence based upon mathematics—the principle of frequencies and harmonics. All business is conducted on the basis of mathematics and all vocations, while perhaps not as directly connected with the numerical as accounting is, all in one way or another have their roots in mathematics.

The beauty of a mathematical demonstration can never be appreciated unless by performing it yourself you can see how each piece fits perfectly in its little groove and the whole scheme coordinates in such a thoroughly fine fashion that it seems to have a distinctive beauty all its own somewhat like a puzzle in which each part adds its bit to the final picture. Mathematics, because of this intriguing beauty has become somewhat of an avocation with some people and is found to be very "mind satisfying," rather like gratifying the appetite. The study of mathematics opens new channels of the mind seemingly unexplored before and one finds oneself thinking and reasoning deeper than ever deemed possible. It is only with the study of geometry or algebra that a person really begins to think things out. This thought developing power of mathematics alone renders it almost invaluable in the present day.

Yesteryear, when one didn't have to have a college degree, or hardly a high school diploma to get a start in life, is past. To have a chance today one has to

* Richard Record is a senior in Central High School at Oklahoma City.—Editor.

be well equipped with a high school certificate, a college diploma, and a purpose. But even without the college degree one of the best ways that a person can prepare himself for the future is by education with plenty of "mathematical backbone." For tomorrow's battles will go to the better equipped man, and the man with a good mathematical background has an excellent weapon.

The War on Euclid

By Charles Salkind
Samuel J. Tilden High School, Brooklyn, New York

To recite the history of the attempts, since even before the turn of the century, to modify the method and content of the Plane Geometry Course or "Euclid," is to invite upon oneself the charge of banality. From the early efforts of Perry and Russell in England, of Laisant in France, of Klein in Germany, of Moore and Hedrick in our own country, to the two most recent reports by the Progressive Education Association and the Joint Commission, through article after article in The Mathematics Teacher and other professional magazines, the battle for reform has been and still is raging.

It is true that in this war against Euclid most classroom teachers of mathematics are noncombatants. Nevertheless, they are vitally interested in its outcome. The vicissitudes of the battle are such that one can discern general trends with the utmost difficulty; most of the time all is confusion. In fact the complexion of the war itself has changed. It is now no longer a question of deposing "Euclid" as the monarch of Plane Geometry: that, it seems to me, has been accomplished. The issue now is: which group of rebels shall succeed to the throne? Shall it be the fusionists, the analytic geometers, the logicians, the intuitionists or experimenters, the transitionists, the correlators, or one of the other groups of reformers? Or shall they all, for the present at least, develop a modus vivendi, with the ultimate claim to rightful jurisdiction to the tenth year put off to a future date?

Into this trying interregnum comes "A Functional Revision of Plane Geometry" by Mr. Nygaard, in the October 1941 issue of The Mathematics Teacher. The article is concisely summarized in the following quotation, taken from the article itself: "In our opinion, then, plane geometry is outmoded—first, with respect to its reliance upon a one-sided deductive type of logic; secondly, because of its abstractions, which disregards applications to the works of man; and, thirdly, from the standpoint of its poor correlation with the methods of arithmetic and algebra."

Surely, no one should take exception to the plea for incorporating into plane geometry instruction, the applications of the mathematical concepts taught to the present and past works of man. The statement "There is probably as much geometry involved in Coulee Dam or the Golden Gate Bridge as in the pages of Euclid" may be termed hyperbolic by some, but the motive prompting the statement is a laudable one. The principle of teaching *applications* is hardly an arguable one today. Present debate centers around the *kind* of applications. Our own school experience has shown that too much concomitant teaching must accompany the *real* applications of geometry and algebra. Pupils—even the bright ones—frequently show an unexpected aversion to these applications; they tax their mentality more than the purely "abstract" problems. Nevertheless, progressive teachers do accept the principle of humanizing their instruction with applications in industry, art, and culture.

A second plea made in the mooted article is for a better correlation of plane geometry teaching with the methods of arithmetic and algebra. In so far as this plea resolves itself into a list of specific suggestions,—such as the use of a single letter to represent the length of a line segment, or the stating of explicit reasons when performing algebraic operations, such as $a+2a=3a$—the question is not sufficiently important to prompt confirmation or denial. For one thing, many teachers do just that very thing. For another, the charge is frequently made that belaboring such "obvious" points as $a+2a=3a$ by writing a reason

for the operation, is the very cause of the de-vitalization of plane geometry.

The suggestion for changing the two-column arrangement for presenting proofs to the essay form of writing, will no doubt stir a hornet's nest. Here the author may be right in saying that inertia always operates to make such reforms difficult to accomplish. But for a large number of mathematics teachers, there is a better reason than inertia for retaining the two-column arrangement. This arrangement was accepted, after a period of agitation, in the interests of clarity. However, the steps in the argument should be properly interrelated by the use of the powerful connectives, "but," "however," "therefore," and so forth. With their aid our immature pupils can the better appreciate that the proof is not a sequence of disconnected steps but a logical chain. The charge that "students are likely to follow its form blindly without giving much thought to the main thread of the argument" can no doubt be made against *both* types of arrangement. Perhaps, the teacher should be free to choose the method to suit the needs of the pupils. Perhaps, both arrangements may be employed, alternatively. Finally, it is possible to use the present bi-columnar plan, interchanged. For example,

1. Since vertical angles are equal

$$1.\ \angle a = \angle b$$

2. But we are given

$$2.\ \angle b = \angle c$$

3. And since a quantity may be substituted for its equal,

$$3.\ \therefore \angle a = \angle c$$

There is also much room for honest differences of opinion on the suggestions contained under the heading, "What can be omitted?" The author states that he is convinced that a number of the theorems dealing with the circle have no future use —for instance, the measurement of all sorts of angles in terms of their intercepted arcs. With this, many will most emphatically disagree. There will also be strong disagreement on eliminating the derivation of the formula for the area of the parallelogram, or of a triangle, or of a trapezoid, after having postulated the area of a rectangle.

The coup de grâce of the article, however, is the charge that plane geometry is outmoded because of its reliance upon a one-sided deductive type of logic. Insofar as a plea is being made for using, in the *teaching* of plane geometry operations intuitive methods for discovering metric properties, we accept the plea, differing, if at all, only in the extent of its use. In fact intuitional, or informal, geometry is quite well established in our junior high schools. In New York City, for example, it is taught in every one of the six grades from 7A through 9B. However, whether this type of geometry *teaching*, call it intuitive or informational or experimental or inductive,[1] precedes the unit of demonstrative geometry or is taught simultaneously with it, it is imperative for us, as purveyors of mathematical Knowledge, to know the *nature* of demonstrative geometry. No one in his enthusiasm for pedagogic reform ought to be guilty of saying, *apropos of the nature of a geometric proof*, "in our opinion the inductive methods ought to be recognized as being fully as valuable as the deductive." There is no excuse for urging the acceptance as a *proof* of the theorem that the sum of the angles of a triangle is 180° a procedure based upon *measurement*, and of befuddling the issue by stating that this theorem "has been questioned by modern mathematicians." On this important point we quote Eric T. Bell, The *Development of Mathematics*, page 4,

"Between the workable empiricism of the early land measurers who parceled out the fields of ancient Egypt and the geometry of the Greeks . . . , there is a great chasm. On the remoter side lies what preceded mathematics, on the nearer, mathe-

[1] The Joint Commission recommends that the word "Informal" be used.—EDITOR.

matics; and the chasm is bridged by deductive reasoning applied consciously and deliberately to the practical inductions of daily life. *Without the strictest deductive proof from admitted assumptions, explicitly stated as such, mathematics does not exist.*

At the risk of being charged with laboring the point, we iterate that intuition and induction have their proper places in the teaching of known mathematics and in inventing new mathematics. We urge only that teachers as well as pupils must realize that without deduction there is no mathematical proof.

Families of Conics with Trigonometric Parameters

H. L. Dorwart
Washington and Jefferson College, Washington, Pennsylvania

1. *Introduction.* The discovery that the coordinates of the ends of major and minor axes of the ellipse whose equation is

$$4x^2+4y^2-4\sqrt{3}\,xy-1=0$$

are

$$\left(\pm\frac{\sqrt{2}+\sqrt{6}}{4},\ \pm\frac{\sqrt{2}+\sqrt{6}}{4}\right)$$

or $\qquad (\pm\cos 15°,\ \pm\cos 15°)$

and

$$\left(\pm\frac{\sqrt{2}-\sqrt{6}}{4},\ \mp\frac{\sqrt{2}-\sqrt{6}}{4}\right)$$

or $\qquad (\mp\sin 15°,\ \pm\sin 15°)$

and hence that the lengths of the semi-major and semi-minor axes are proportional to cos 15° and sin 15°, has led me to consider the nature of conics whose semi-axes are trigonometric functions. In this paper, a few of the simpler and more interesting one parameter families of such conics will be discussed. The general families are all well known, but the specialization of the parameter leads to some simple constructions that may be of interest to both teachers and students of analytic geometry.

2. *Conics touching four lines.* The family of ellipses

$$(1)\qquad \frac{x^2}{\cos^2\theta}+\frac{y^2}{\sin^2\theta}=1$$

can be represented by

$$(2)\qquad \frac{x^2}{1-\lambda^2}+\frac{y^2}{\lambda^2}=1$$

for $\lambda=\sin\theta$, with $0°<\theta<90°$. This family has an envelope which can be found by the usual methods, and which consists of the four lines joining the points (1, 0), (0, 1), (−1, 0), (0, −1) in order. Hence (1) represents the ellipses lying within and touching the square formed by these lines.

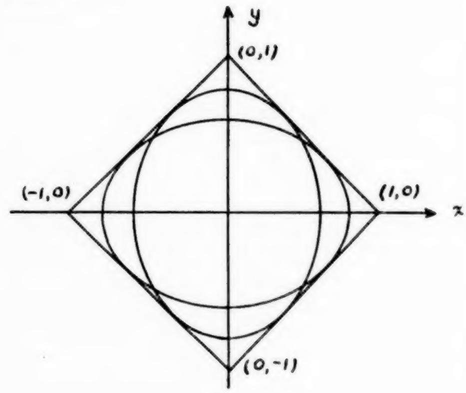

Fig. 1

Since the director circle (the locus of points of intersection of two perpendicular tangents) for the ellipse $x^2/a^2+y^2/b^2=1$ is $x^2+y^2=a^2+b^2$, each member of (1) has the same director circle, namely the unit circle. The angle θ is the inclination of the line joining the origin to the point P on the unit circle which projects orthogonally on the x and y axes into the ends of the semi-axes. The point of contact of (2) with $x+y-1=0$ is $(1-\lambda^2, \lambda^2)$ or $(\cos^2\theta, \sin^2\theta)$. Hence the inclination α of the line joining the origin to the point of contact is given by

$$\tan\alpha=\tan^2\theta$$

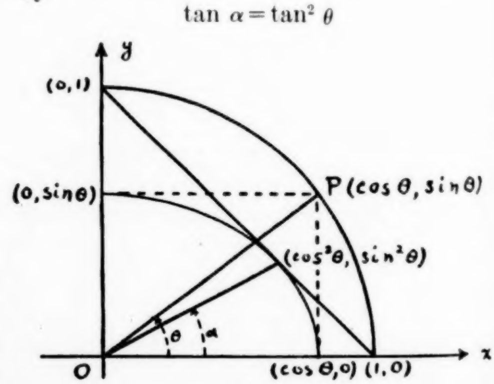

Fig. 2

Evidently as $\theta \to 0$, $\alpha \to 0$; as $\theta \to 90°$, $\alpha \to 90°$; and when $\theta = 45°$, $\alpha = 45°$. The determination of the values of θ for which $|\theta - \alpha|$ is maximum involves interesting calculus, algebra and trigonometry.

$$|\theta - \alpha| = |\theta - \text{arc tan } \tan^2 \theta|,$$

and the derivative of this expression with respect to θ equated to zero is

$$1 - \frac{2 \tan \theta \sec^2 \theta}{1 + \tan^4 \theta} = 0$$

or

$$\tan^4 \theta - 2 \tan^3 \theta - 2 \tan \theta + 1 = 0.$$

Descartes' rule of signs informs us that there will be no more than two positive real roots, no negative real roots, and hence at least two imaginary roots. If there are two real positive roots, they are evidently reciprocals of each other. Following the usual procedure for a reciprocal equation in standard form, divide through by $\tan^2 \theta$ and change the order of terms to obtain

$$(\tan^2 \theta + \cot^2 \theta) - 2(\tan \theta + \cot \theta) = 0.$$

Now let

$$z = \tan \theta + \cot \theta = 2 \operatorname{cosec} 2\theta.$$

Then

$$\tan^2 \theta + \cot^2 \theta = z^2 - 2$$

and the equation becomes

$$z^2 - 2z - 2 = 0.$$

Hence

$$z = 1 \pm \sqrt{3} = 2 \operatorname{cosec} 2\theta.$$

For a real solution, the minus sign has to be discarded and

$$\theta = 23° \; 31.8', \qquad 66° \; 28.2'$$

both of which make $|\theta - \alpha|$ a maximum.

Rational values of the semi-axes are given by

$$\cos \theta = \frac{2m}{1 + m^2}, \qquad \sin \theta = \frac{1 - m^2}{1 + m^2}$$

for rational m.

Also, equation (2) represents the same family of ellipses for

$$\lambda = \tanh \beta, \quad \sqrt{1 - \lambda^2} = \operatorname{sech} \beta, \quad 0 < \lambda < 1$$

where β is the inverse gudermannian of θ, i.e. $\beta = gd^{-1}\theta$.

But equation (2) with λ unrestricted, represents hyperbolas as well as ellipses. It is, in fact, the well known family of conics touching four lines.*

Writing (2) in the form

$$\frac{y^2}{\lambda^2} - \frac{x^2}{\lambda^2 - 1} = 1$$

and placing $\lambda = \sec \phi$ (or $\operatorname{cosec} \phi$) with $0° < \phi < 90°$, the hyperbolas

$$\frac{y^2}{\sec^2 \phi} - \frac{x^2}{\tan^2 \phi} = 1$$

are obtained. The point of tangency S with $x - y + 1 = 0$ in the first quadrant is $(\lambda^2 - 1, \lambda^2)$ or $(\tan^2 \phi, \sec^2 \phi)$. The slopes of the asymptotes are $\pm \operatorname{cosec} \phi$ and the interpretation of ϕ is shown in figure 3 in which $AQ = OR = \operatorname{cosec} \phi$.

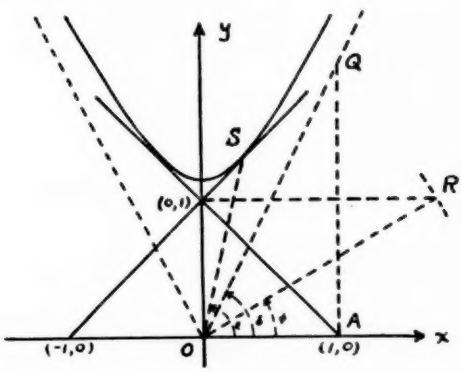

Fig. 3

Hyperbolic functions can be used by taking $\lambda = \cosh \gamma$ (or $\operatorname{ctnh} \gamma$) where $\gamma = gd^{-1}\phi$.

3. *Confocal Conics.* The family

$$(3) \qquad \frac{x^2}{\sec^2 \psi} + \frac{y^2}{\tan^2 \psi} = 1$$

* E.g., see Conic Sections by C. Smith, §286, p. 366.

with $\lambda=\sec\psi$ and $0°<\psi<90°$, becomes

$$(4)\qquad \frac{x^2}{\lambda^2}+\frac{y^2}{\lambda^2-1}=1$$

and represents confocal ellipses with foci $(\pm 1, 0)$. The eccentricity is $1/\lambda=\cos\psi$, and as $\lambda\to\infty$, $e\to 0$, so that the limiting case of the ellipses is the circle of infinite radius. The coordinates of the ends of the focal chords are $(\pm 1, \pm\sin\psi\tan\psi)$, and the slope of OP is $\sin\psi$. The angle ψ and the ends of the focal chords can be constructed very simply by ruler and compass as follows (see figure 4).

Draw OP. Then $FQ=\sin\psi$. Draw a horizontal line through Q, and on this line locate R such that $OR=1$. Angle ψ is now determined, and if OT is taken equal to $FQ(=\sin\psi)$, then $TL=\sin\psi\tan\psi$ and the horizontal line through L cuts FQ extended in M, an end of a focal chord.

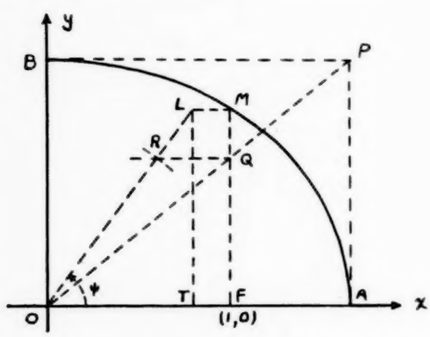

FIG. 4

Rewriting (4) in the form

$$\frac{x^2}{\lambda^2}-\frac{y^2}{1-\lambda^2}=1$$

and taking $\lambda=\cos\zeta$ with $0<\zeta<90°$, we have confocal hyperbolas with foci $(\pm 1, 0)$. The eccentricity is now $\sec\zeta$, the ends of the focal chords are $(\pm 1, \pm\tan\zeta\sin\zeta)$, and ζ is the inclination of an asymptote. The construction of the ends of a focal chord is apparent from figure 5, where $FQ=OL=\tan\zeta$ and $TL=FM=\tan\zeta\sin\zeta$.

4. ***Hyperbolas whose asymptotes approach a limiting position.*** The family

$$(5)\qquad \frac{x^2}{\tan^2\xi}-\frac{y^2}{\sec^2\xi}=1$$

is not covered by the discussion previously given. For $\lambda=\sec\xi$, $0°<\xi<90°$, it becomes

$$(6)\qquad \frac{x^2}{\lambda^2-1}-\frac{y^2}{\lambda^2}=1.$$

This family of hyperbolas is not confocal nor does it have an envelope. However it

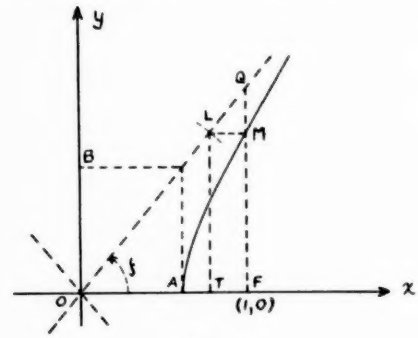

FIG. 5

does have the property that as $\lambda\to\infty$ the slope of the asymptotes, $\pm\operatorname{cosec}\xi$, approaches ± 1. The ellipses of this family are the imaginary ones

$$\frac{x^2}{1-\lambda^2}+\frac{y^2}{\lambda^2}=-1.$$

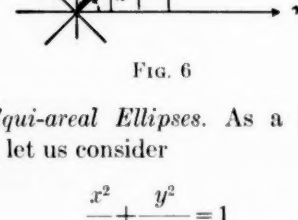

FIG. 6

5. *Equi-areal Ellipses.* As a final example, let us consider

$$(7)\qquad \frac{x^2}{\lambda^2}+\frac{y^2}{1/\lambda^2}=1$$

where

$$\lambda=\sin\mu(\cos\mu)$$

or

$$\lambda = \sec \nu (\operatorname{cosec} \nu)$$

or

$$\lambda = \tan \rho (\cot \rho).$$

This familiar family of ellipses* has for envelope the hyperbolas $xy = \frac{1}{2}$, $xy = -\frac{1}{2}$, and has the further property that the area of each member $= \pi$ square units.

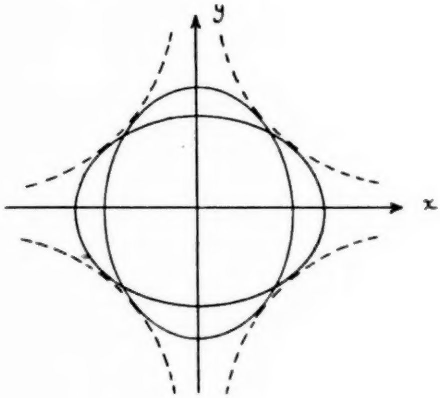

FIG. 7

The vertical ellipses are given by $\lambda = \sin \mu$ ($\cos \mu$) and the horizontal ones by $\lambda = \sec \nu$ ($\operatorname{cosec} \nu$), or all members by $\lambda = \tan \rho$ ($\cot \rho$). The point of contact in the first quadrant with the envelope is $[(\sqrt{2}/2)\lambda,\ (\sqrt{2}/2)(1/\lambda)$ or $(\lambda \sin 45°,\ 1/\lambda \sin 45°)$ which suggests a simple construction for the point of contact. Draw a line bisecting the second and fourth quadrants. The projections of the semi-axes on this line give the coordinates of the point of contact.

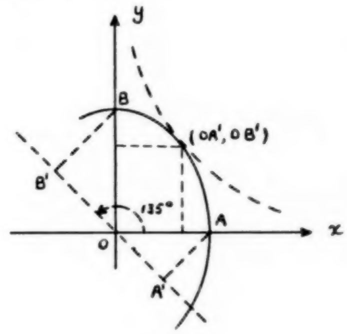

FIG. 8

Since the slope of the line through the origin and the point of contact is the same as the slope of the line through the origin and the point P which projects into the ends of the semi-axes, these lines coincide. Hence if the envelope is already drawn, the point of contact is determined by the intersection of OP with the envelope.

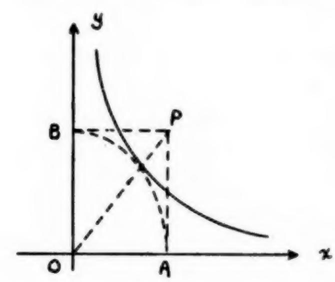

FIG. 9

* See e.g. Hyperbolic Functions, Smithsonian Math. Tables, Introduction, p. XX.

The Sphere

The sphere is Nature's favorite form
Which giant suns approximate,
And balls of lightning ride a storm
As toward the earth they gravitate.

And Nature's great economy.
—Least surface for the volume wrapped—
Throws all the plastic bodies she
Designs, so they this law adapt.

From mass, infinitesimal
To greatest island universe,
A drop of mercury let fall
Or hail which summer clouds disburse:

All take a form, near spherical
When texture lends to moulder's art,
Cosmic or atmospherical
They ride in Nature's rotund cart.

OLA ESKELSON

Mathematics for All*

By Harl R. Douglass
Director of the College of Education, University of Colorado, Boulder, Colorado

This is a mathematical world. Everything we do seems related, in some way, to measurement. For a thousand years the world has been growing more and more mathematical. For a thousand years measurement and computation has been playing an increasingly significant part in the lives of everyone. The tendency to become at once the creatures and the masters of precise quantitative thinking and action is a rapidly accelerating phenomenon.

A variety of factors, social and otherwise, have contributed to this tendency. Developments in transportation have played their part in many ways—time tables, the mathematics of construction, repair, purchasing, and operating of automobiles. Specialization in agriculture and other vocations have increased enormously the volume of business transactions—the selling of labor, farms and other products, and the purchasing of practically everything for consumption in the home. And finally, the present crisis has revealed the importance, in total war, of quantitative literacy in the citizen.

The self-sufficient home has disappeared and in its place has come a producing and a purchasing unit which is compelling us to think in precise, quantitative terms and manners. The housewife of 1880 might well think of food in approximate terms of a hundred or so jars of fruit, seven or eight hams, fifteen or twenty hens, and a cow. She was under no necessity of calculating what was the best buy. Only for a small portion of her purchases was she compelled to make decisions based upon prices tacked on shelves for items of food of various sizes. The prices, as a matter of fact, were not tacked on the shelves or otherwise made known to her unless she asked. Cooking and recipes were vaguely or less exact in so many eggs, so many cupfuls, so many tablespoonfuls and the like, but weighing had not yet arrived and scales were rarely found in the kitchen of 1880.

In like manner other developments in communication, in the knowledge of diet and other phases of health, in commercialized amusement and recreation, in artistic and aesthetic fields, in installment buying have actually increased the need for present quantitative thinking.

In the realm of social security; in fire, life, hospital, medical, and automobile insurance; in annuities and other forms of providing for the future; in new forms of taxes and the extension of old forms that make democracy indeed a cooperative form of spending, our need for computational thinking has increased many fold.

Problems of travel did not require the quantitative treatment that they do today. The dashboard boasted no array of mathematical gauges to measure this, that, and the other, as does the instrument panel of automobiles and airplanes. There was no calculation of how many miles to the bushel of corn or to the bale of hay. In the construction, repair, and upkeep of buggies and wagons, measurement in thousandths of an inch was not dreamed of.

In an interesting paradox to this very marked increase in the mathematical needs of all of us, there has been a constantly decreasing percentage of children in school above the eighth grade studying mathematics. It is worth noting that in 1910 more than three-fourths of all high school students were enrolled for a class in mathematics while in 1940 the proportion of high school pupils enrolled for a class in mathematics is less than one in

* Read at the Annual Meeting of The National Council of Teachers of Mathematics at San Francisco, February 20, 1942.

three. *What a cock-eyed world it is in which, as the needs for mathematics becomes greater and more universal, the smaller is the percentage of high school students studying it.*

There are some good reasons why these inconsistent trends have occurred. In the first place, the numbers of boys and girls going through high school have doubled (approximately) every ten years since 1880. Whereas in 1880 only one in seventy, in 1900 only one in twenty, and in 1920 only one in five youngsters of high school age attended high school—in 1940, the rate was seven in ten. The high school curriculum of 1880 and through 1920 was avowedly decorative, disciplinary, and college preparatory in function. It was not intended to minister to the life needs of the majority of future citizens. *Mathematics as we have taught it in the high school has been confined largely, in its practical values, to the vocational needs of a small group of college trained workers—chiefly engineers.*

Mathematics as taught in the high school has had little to contribute to the vocational needs of business men, to most types of skilled workmen, farmers, or to the common laborer. Workers in these and other fields have found that their mathematical needs called for arithmetic, intuitive and constructive geometry, and occasionally the simplest algebra. They were supposed to have learned these in the elementary school, but if learned, they had in great part been forgotten. When needed they had to be relearned and extended.

The mathematical needs of the home—diet, economical purchasing, budgeting, social security, transportation, etc.—these did not call for simultaneous equations, the proving of geometric theorems, or for trigonometry. Rather, they called for reasoning and accuracy in the use of arithmetic, of intuitive geometry, and of formulae of the simple type.

The mathematics taught in the elementary school has indeed been of the type needed, in practice it has proved inadequate. The children there are too young, too immature. They have too little experience to understand or to be interested in the life problems calling for mathematics. Too little time has been available to result in mastery. The applications are too complicated for twelve year olds. We see this when seventh graders attempt to calculate the comparative returns from various kinds of investments, or whether it is cheaper to buy a house at so much in a city where taxes are on the average so much—with repairs and depreciation likely to be so much, etc.—or to rent one of similar size, etc., at so much a month.

It seems quite illogical to attempt to meet the mathematical needs in the elementary school, and then to confine instruction in mathematics above the elementary school to a type of mathematics which, is designed for a special group going on into further study of mathematics or science, or into engineering in college. At a time when the great majority of students did not go beyond the elementary school, it was probably defensible to crowd into the grades the mathematics needed in everyday life. *But with the great increase in the number going on to high school, the necessity for the premature introduction of difficult arithmetical calculations disappeared and appropriate adjustments are called for.*

Today when only one high school freshmen in ten will ever finish a year in college, and one high school senior in five will ever enter college, the failure to offer in high school mathematics other than algebra, geometry and trigonometry is not defensible from the point of view of the student nor from that of the best interests of mathematics. Even the college-going student needs a much greater mastery of arithmetical processes—particularly ratios and proportion, and computations, operations with common and decimal fractions —as practically every professor of chemistry, physics, accounting, and engineering will testify.

The arbitrary location of all arithmetic

in the first eight grades, and only algebra, geometry and trigonometry in the last four grades conforms neither to the educational needs of the present age nor to what we know of psychology—of the abilities and interests of young people of the ages concerned.

Many high school teachers of mathematics would, it is true, find it necessary to review and extend their knowledge of arithmetic and to acquire much more knowledge concerning the applications of arithmetic to various phases of life. Yet this does not seem an unfair burden to ask teachers to assume this in the interests of the pupils and the nation they purport to serve.

Perhaps as influential as any other force is the questionable attitude taken by many teachers of mathematics that it would be beneath their dignity as majors in mathematics—students of calculus and differential equations, to teach mathematics other than algebra, geometry, and trigonometry in high school. I can well appreciate this position, its naturalness and its dangers. When I graduated from the University, having majored in mathematics and with the innocence and igorance of life outside the classroom typical of young graduates, I, too, wished to demonstrate my knowledge of "higher" mathematics and my ability to teach it.

I should not like to have anyone jump to the absurd conclusion that there is no place in my thinking for the traditional college preparatory courses in mathematics in the secondary school. It is my firm conviction that a very considerable fraction—maybe in some schools half, should have a year of algebra and perhaps a fourth should have geometry, more algebra and trigonometry. In all but the very large high schools such courses should not be offered before the tenth grade and in the small schools, let us say those with enrollments of less than 300, they should begin in the eleventh grade, since in these smaller schools there are rarely enough pupils going on beyond two years of mathematics to justify giving more. In all schools, *abler students in the eleventh and twelfth grades who have not had a regular course in algebra should be urged to enroll for one*. The present war developments show clearly that for the purposes of war, at least one male high school graduate in five or 10% of all should have algebra and trigonometry.

There should, in addition to the algebra-geometry-trigonometry sequence be developed in the high school another series of offerings in mathematics for the great majority of pupils. This should be composed largely of arithmetic and its applications to all phases of life, home, shop, farm, business, health, travel and transportation, social security, etc. It should also include considerable geometry of construction and measurement of surfaces and volumes. It should also include, correlated in its applications with arithmetic and geometry, at least six or seven months of algebra, literal and negative numbers, much work with simple equations and formulae, a large variety of applications particularly to science, considerable work with graphical representation, and some work with simpler statistical constants and procedures.

This sequence for the general school population might well be organized in a variety of ways. Among them might be mentioned the following in addition to which there should be offered at least one year of algebra and one year of geometry or one year of algebra and trigonometry—both in the larger schools.

Plan A

Grades eight through twelve inclusive. One semester each year or two or three hours a week through the year—including some algebra in each year especially in grades eight through ten, and required in all grades, except 12, of all pupils not taking other mathematics. One additional elective of applied mathematics should probably be offered in grade twelve of the type indicated under Plan B.

Plan B

(Based upon two years of arithmetic in grades 7 and 8).

Tenth grade—one year (required)—largely applied arithmetic, formulas and simplest algebra.

Eleventh and twelfth grade—two or three elective offerings of one semester each—two of the three being required of all pupils not taking other mathematics.

a. Mathematics of business
b. Mathematics of science and shop
c. Mathematics of home and consumer

Plan C

The expansion and enrichment of mathematics now taught in grades 7 and 8 into a three year course required in grades 7, 8, and 9. Grades 11 and 12 as under Plan B.

In any one of these plans the pupil will have had three or four semesters of general mathematics above the seventh grade before reaching the eleventh grade. Those who take two years of college preparatory mathematics in grades eleven and twelve would, by reason of increased maturity and ability and by reason of having some introduction to algebra and geometry, be able to complete second and third semester algebra in the eleventh grade and either plane and solid geometry or one semester of geometry and one of trigonometry in the twelfth grade. In larger schools this college preparation sequence should be expanded to three years.

It will be noted that in these plans more mathematics is required than is the case in most schools today. There are two reasons why this should be so:

1. More mathematics is needed in the life of Americans of this day.
2. In the elementary school, mathematical topics having been crowded too prematurely in the lower grades are being shifted at least a grade upward. The congestion in grades seven and eight requires relief which can be given only in one of two ways:
   a. Leaving out considerable of that previously taught in those grades.
   b. Transferring to grade nine or later, topics previously attempted in grades seven and eight.

The two courses in ninth grade mathematics offered in some schools do not meet the needs of the situation. In too many cases both are really courses in algebra. *There are needed two essentially different types of sequence in high school mathematics.* One is a required sequence which meets the mathematical needs of all as they will find them in business, home, social security, health, travel, and transportation, etc. The other is a technical sequence in algebra, geometry, and in all larger schools, trigonometry—for those who will engage in any type of engineering or scientific pursuit—essentially a pre-vocational course.

Developments along the lines of this dual sequence are certain to come—in fact are already on the way. Many junior high schools are offering three years of required mathematics. In other schools a semester or a year of non-college preparatory mathematics in the senior high school is being offered. The chief obstacles to the more rapid adaption of the curriculum in mathematics to the needs of the day are (a) schoolmarms, male and female, who have lived only a sheltered academic existence and who are relatively innocent of the details of the uses of mathematics in the daily lives of all the people, and (b) college professors who live in ivory towers and who say of the great mass of future American citizens who have come into high school since the present generation of college professors graduated and whom they know not well or understand—"If they have not bread, let them eat cake." But although normally slow to result in adequate action, public opinion and teachers of good common sense will eventually force these types of school people as already has occurred in many other educational areas, to do what they should have done on their own initiative.

The present emergency, throwing, as it does, the spotlight on the importance of mathematics in all forms of cooperative or technical enterprise, offers us an opportunity not only to recover lost ground, but to establish the importance of our field in a realistic way that cannot be denied, thus relieving us of the constant necessity of apologizing for our failure to educate for the mathematical needs of the great mass of American men and women.

THE ART OF TEACHING

Teaching Logarithms

By J. P. Harper, *University of Scranton, Scranton, Pennsylvania*

The importance and utility of logarithms cannot be denied by anyone who has any familiarity with them. However, the extent to which logarithms are used does not at all seem to compare to the advantage they hold over other methods of computation. Not many teachers of chemistry and physics, for example, in the colleges encourage their students to use logarithms where they would be of such great advantage as a time saver. Many students use logarithms in these courses but many more do not as they lack confidence in their ability to handle them and lack an understanding of the real significance of logarithms and hence continue to plod laboriously through the tedious processes of arithmetic division, multiplication and extraction of roots. In teaching such a course as physics most teachers give their students numerous numerical problems to solve without the use of logarithms. The students spend an unjustified amount of time on the arithmetic processes and may thereby lose sight of the real significance of the problem. Therefore, I feel that more emphasis could be placed on the study and application of logarithms and that more encouragement could be given to the use of logarithms in such college courses as chemistry and physics. This is especially true in the Arts College, but does not apply to the same extent to the Engineering and Technical Colleges as much more importance is attached to the study and use of logarithms early in the student's college career. To sum up then, it can be suggested that in the study of logarithms an attempt should be made to make them more impressive, more time should be spent on the real significance of logarithms, and that a more thorough explanation of the significance and use of the tables should be given. Also it can be urged that a much wider use should be made of logarithms in such courses as chemistry and physics where they would prove to be a great time saver.

In making logarithms more impressive and to give the student a fuller conception of their real meaning the first thing to be considered and continually stressed throughout the study of logarithms is that the logarithm of a number (common log) is really the power to which the base 10 is raised to give the number. Thus, associated with the logarithm equation $\log_{10} N = X$, one should always have before him the corresponding exponential equation $N = 10^x$. This then makes it imperative that the study of logarithms be preceded by a thorough study of exponents. In fact it is not feasible to present the subject of logarithms without first presenting the subject of exponents. This point is sometimes neglected by those who teach logarithms.

With the exponential idea firmly in mind it is a simple matter to derive the rules for the use of logarithms; that is, if with every logarithm equation we always write out the corresponding and equivalent exponential expression. In deriving the product logarithm law, for example, we would proceed somewhat as follows:

Let $\log M = X$, also expressed as $M = 10^x$

and $\log N = Y$, also expressed as $N = 10^y$.

Then $\log M \cdot N = \log 10^x \cdot 10^y$

$= \log (10^{x+y}) = X + Y$

and $\log M \cdot N = \log M + \log N$.

A similar procedure is followed in deriving the quotient and exponential rule—the exponential rule combining powers and roots. It is also important to stress the fact that $\log 10^x = X$. From the very definition of the logarithm of a number the logarithm of 10 to a power is that power. From this follows the important point that $\log 100 = 2$, $\log 1000 = 3$, $\log .001 = -3$ etc.

Concerning the logarithm tables, students sometimes fail to get firmly fixed in mind the fact that the numbers in the number column only extend from 1 to 10 and that there should be decimal points after the first digits in the numbers in this column. Thus, in a table the number 432 in the number column really means 4.32 etc. Also in some logarithm tables the decimal point of the mantissa is not put in. This leads to confusion if it is not pointed out in the very beginning that the logarithm numbers require a decimal before them. Having made these points clear it will be easy to put across the idea of characteristics, for before looking up the mantissa of a number in a table it must first be reduced to a number between 1 and 10 which can be found in the table. Thus, if one were looking up the logarithm of 436.5 it would first have to be thought of as 4.365×10^2 so as to have a number that would be found in the table; then

$$\log 4.365 \times 10^2 = \log 4.365 + \log 10^2 \quad \text{(by product rule)}$$
$$= \log 4.365 + 2$$

and $\log 436.5 = 2.63998$.

The 2, or characteristic part of the logarithm, which is always a whole number, is placed first and the fractional part of the logarithm, or mantissa, which is found in the table is placed last. In a similar way,

$$\log .04365 = \log 4.365 \times 10^{-2} = .63998 - 2,$$

which is written, $\log .04365 = \bar{2}.63998$.

The —2 has the minus sign above it so as not to make the entire logarithm negative. In the above we see the reason for the rule for finding the characteristics. Since we must move the decimal point to a position to the right of the first significant number so as to have a number for which we can find the logarithm in the table, this means that we must multipy by 10 to the appropriate power as shown in the example above in order not to change the original value. Making use of the product rule for logarithms we find that the 10^x adds the whole number X to the mantissa. X is, of course, equal to the number of places the decimal has to be moved—if the decimal is moved to the left, the characteristic is positive and if it is moved to the right the characteristic is negative. A convenient wording for the "characteristic rule" found in most texts is somewhat as follows: starting to the right of the first significant figure of the number, for which the logarithm is to be found, count to the right or left to the decimal point and this gives the value of the characteristic part of the logarithm, positive or negative respectively.

Some students are curious as to how the logarithm tables are made up, that is, where the numbers come from. I find it stimulates interest in the study of logarithms and makes the subject more impressive to give the students a method of finding logarithms to three or four digits. The method is only an approximation (derived from differential calculus) but gives surprisingly accurate results. To show that one can start from "scratch," so to speak, and build up a logarithm table of any desired size is usually a source of surprise and interest to the students. A teacher might introduce this idea by presenting a hypothetical situation in which one found himself without a log book and found it absolutely necessary to find a few logarithms. Below is presented a method of building up a log table. The values are determined to five digits although the values are not all accurate.

The approximation formula for finding logarithms can be written in the form

$$\log (N+X) = \log N + \frac{X\ .4343}{N + \dfrac{x}{2}}.$$

Here N is a number for which the logarithm is known, $N+X$ is a number for

which the logarithm is desired, and X is the increment between these numbers. The .4343 represents the slope at any point N of the logarithm curve [log N (ordinate) plotted against N (abscissa)] multiplied by the number N. Also the number .4343 is $\log_{10}e$, since slope of the curve is

$$m=\frac{d}{dN}\quad \log_{10}N=\frac{\log_{10}e}{N}$$

and $Nm=\log_{10}e$. This value $\log_{10} e$ can actually be found to three digits by plotting a curve with the values given below in Table I, finding the slope at a few points, then multiplying these slopes by the corresponding numbers N and averaging the results as shown in Fig. 1.

To build up a logarithm table and to use the equation one must either take some known logarithms, or better still, extract

TABLE I

$10^{1/2}=3.1623$, or $\log 3.1623=.500$

$10^{1/4}=\sqrt{10^{1/2}}=1.7785$, or $\log 1.7785=.250$

$\sqrt{10^{1/4}}=10^{1/8}=1.3335$, or $\log 1.3335=.1250$

$10^{3/4}=10^{1/2}\cdot 10^{1/4}=5.6234$, or $\log 5.6234=.750$

$10^{3/8}=\sqrt{10^{3/4}}=2.3714$, or $\log 2.3714=.375$

$10^{5/8}=10^{1/4}\cdot 10^{3/8}=4.2170$, or $\log 4.2170=.625$

$10^{7/8}=10^{3/8}\cdot 10^{1/2}=7.4990$, or $\log 7.4990=.875$

$10^{7/16}=\sqrt{10^{7/8}}=2.7384$, or $\log 2.7384=.4375$

$10^{15/16}=10^{1/2}\cdot 10^{7/16}=8.6596$, or $\log 8.6596=.9375$

size and accuracy a good value of the $\log_{10}e$ can be found from it. The next step is to obtain from the formula and the table above the logarithms of the whole numbers from 1 to 10. Starting from 3, the logarithm can be obtained by using log 3.1623 = .5 and the formula. We would have,

$$\log 3=\log 3.1623-\frac{.1623\times .4343}{3.0811}$$

$$=.5-.02288=.47712.$$

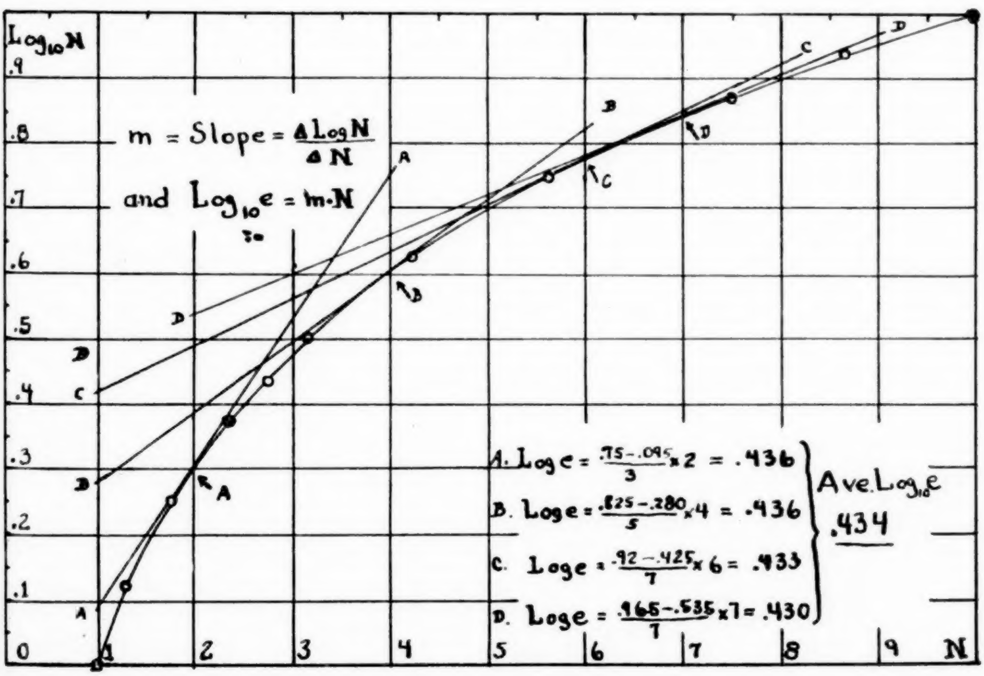

FIG. 1. Logarithm Curve ($\log_{10}N$ vs N)

various roots of 10 for the reference logarithms; for example, we can get $\sqrt{10}$, or $10^{1/2}$, $10^{1/4}$, $10^{3/4}$ etc. Thus, we could build up Table I.

From these values the curve of Fig. 1 can be drawn. If this curve is of sufficient

Next we can get log 7.5 from log 7.4990 = .875, and by using the formula,

$$\log 7.5=\log 7.4990+\frac{.001\times .4343}{7.4995}$$

$$=.875+.00006=.87506.$$

Now, $\log 7.5 = .87506$,

and $\log 2.5 = \log \dfrac{7.5}{3} = .87506 - .47712$,

or $\log 2.5 = .39794$;

also $\log 25 = 1.39794$,

and $\log 5 = \frac{1}{2}(\log 25) = \dfrac{1.39794}{2}$

$= .69897$.

The value of log 2 follows from log 5: $\log 2 = \log 10/5 = 1 - .69897 = .30103$.

Having found the logarithms of 2 and 3 we can then get $\log 4 = 2 \log 2$, $\log 8 = 3 \log 2$, $\log 6 = \log 2 + \log 3$, and $\log 9 = 2 \log 3$. This leaves log 7 of the numbers from 1 to 10.

$$\log 7 = \tfrac{1}{2} \log 49 = \tfrac{1}{2}\left(\log 50 - \frac{1 \times .4343}{495}\right)$$

$$= \tfrac{1}{2}(1.69897 - .00877)$$

$$= \tfrac{1}{2}(1.69020) = .84510.$$

After completing this determination of the logarithms of these basic numbers we can then proceed to the determination of the logarithms of the other numbers. In Table II, given below, we have the logarithms of numbers from 1.0 to 10.0 at intervals of 0.1. Many of the numbers are multiples of the basic numbers in the first column and are filled in first (such as $7 \times 8 = 56$, $9 \times 8 = 72$, etc). Then to get the other values with the accuracy shown the procedure is to start at the top, find the value of log 9.9 and from this the values of log 1.1, log 2.2 etc.; then the value of log 9.8 and the logarithm of such factors of 9.8 as 4.8, 2.4 and 1.2. Thus, going on down the line we find that the formula is used only a relative small number of times; in fact it was used about 40 times (some of these being checks). After the table is made up it can be used with accuracy in conjunction with the formula for finding logarithms of four or five digit numbers such as 3.9463 or 9.1758, etc. For example,

$$\log 3.9643 = \log 3.9 + \frac{.0463 \times .4343}{3.9231}$$

$$= .59106 + .00513 = .59619.$$

Similarly the logarithms of any other numbers not in the table can be found.

Table II has been compiled to five

TABLE II

N	0	1	2	3	4	5	6	7	8	9
1	.00000	(9.9/9) .04140	(4×3) .07918	(6.5/5) .11394	(5.6/4) .14613	(3×5) .17609	(8×2) .20412	(6.8/4) .23045	(3.6/2) .25527	(9.5/5) .27875
2	.30103	(3×7) .32222	(2×1.1) .34243*	(9.2/4) .36173	(2×1.2) .38021	(7.5/3) .39794	(2×1.3) .41497	(3×9) .43136	(5.6/2) .44716	(5.8/2) .46240
3	.47712	(9.3/3) .49136	.50515	(3×1.1) .51852*	(6.8/2) .53148	(7×5) .54407	(6×6) .55630	(7.4/2) .56820	(2×1.9) .57978	(3×1.3) .59106
4	.60206	.61278	(2×2.1) .62525	.63347	(4×1.1) .64346*	(9×5) .65321	(9.2/2) .66276	(9.4/2) .67210	(9.6/2) .68124	(9.8/2) .69020
5	.69897	.70757	.71600	.72427	(9×6) .73239	(5×1.1) .74037*	(7×8) .74819	(3×1.9) .75587	.76342	.77085
6	.77815	.78533	(2×3.1) .79239	(3×2.1) .79934	(8×8) .80618	.81291	(6×1.1) .81955	.82608*	.83251	.83885
7	.84510	.85126	(2×3.6) .85733	.86332	.86923	.87506	.88081	(7×1.1) .88650*	.89210*	.89736
8	.90309	.90849	.91381	.91908	(4×2.1) .92428	.92942	.93450	.93952	(8×1.1) .94449*	.94939
9	.95424	.95904	.96379	.96848	.97313	.97772	(6×1.6) .98227	.98678	.99123	.99564
10	.00000									

places. This was on the basis of knowing the value of $\log_{10}e$ to four significant figures. From the curve of Fig. 1 it is hard to obtain it to any greater accuracy than three digits. In this case the logarithms could not be calculated with accuracy beyond the 4th digit.

One further point on the use of the formula might be mentioned. Unless the increment x between the number for which the logarithm is known and the number for which the logarithm is desired is relatively small by comparison to the numbers then the last digit in the logarithm is likely to be in error. For example: If we wanted the log 1.1 we might try the formula and get,

$$\log 1.1 = \log 1 + \frac{.1 \times .4343}{1.05} = 0 + .04136,$$

whereas the actual value is .04139. But an increment equivalent to .1 can be used for numbers above 2. For example

$$\log 2.1 = \log 2 + \frac{.1 \times .4343}{1.05} = .30103$$
$$+ .021185 + \log 2.1$$
$$= .30103 + .02119 = .32222.$$

And for numbers close to 10 even increments as large as .2 and .3 can be used without an appreciable error—as the curve is very flat in that region.

After making up Table II without reference to another logarithm table the values were checked and it was found that a few of them were in error (those marked by *) in the last place. Although this is not to be considered serious an effort was made to correct them. It will be noticed that the logarithm of 1.1 is given as .04140 but it is given as .04139 in other tables. Also all the multiples of 1.1 except 9.9 calculated by the formula from 10 are in error by 1 in the last place. To correct these errors a more accurate value of 1.1 was determined from 1.21 as follows:

$$\log 1.21 = \log 1.20 + \frac{.01 \times .4343}{1.205}$$
$$= .07918 + .00360 = .08278$$

$$\log 1.1 = \frac{\log 1.21}{2} = \frac{.08278}{2} = .04139.$$

With this correction for log 1.1 all the other logarithms of multiples of 1.1 can be corrected. In the same way the few other logarithms which are in error could be corrected; i.e. by "cross checking" and by the use of the formula.

It might also be of interest to note that in the same way that the formula is used to find logarithms a corresponding equation can be used to determine antilogs to a high degree of accuracy. If we take the logarithm formula

$$\log (N+X) = \log N + \frac{X\ .4343}{N + \frac{x}{2}}$$

and solve for the value of X we get

$$X = \frac{(\log (N+X) - \log N)N}{.4343 - \frac{1}{2}(\log (N+X) - \log N)}$$

or

$$X = \frac{D \cdot N}{.4343 - \frac{1}{2}D}$$

where D is the difference between the given logarithm for which we wish to find the antilog and the nearest logarithm to this value. As an example: Suppose we desired to find antilog .55813. The nearest logarithm to this is log 3.6 = .55630, thus

$$D = \log (N+X) - \log N = .00183, \quad N = 3.6$$

and

$$X = \frac{.00183 \cdot 3.6}{.4343 - .000915}.$$

Then, $N + X = 3.6 + .01519 = 3.61519$.

From a logarithm table we find antilog .55813 = 3.61518. In either case we would write these values as 3.6152 (correct to five digits).

The above suggestions have been tried in college freshman mathematics classes with good results and justify the slightly greater amount of time spent on this important topic. The author, therefore, wishes to pass on the above expressed ideas to others for whatever they may be worth.

The National Council of Teachers of Mathematics

Eighth Summer Meeting with the N.E.A.*

June 29-30, 1942, Denver, Colorado

Headquarters: Hotel Argonaut

Rates
Single rooms with bath $2.50.
Double rooms-Double bed with bath $5.00.
Double rooms—Twin beds with bath $6.00–$8.00.
General Theme: Mathematics in These Times.

Program

Monday, June 29, 1942, 1:45 P.M.

Joint Session with the Department of Secondary Education
The Secondary School Today
3:15 P.M. General Session of the National Council
Presiding: R. L. Morton, First Vice-President, Ohio University, Athens, Ohio
Mathematics and Youth—Lon Edwards, Colorado State College of Education, Greeley, Colorado
Some Problems that Cast Long Shadows into the Future—Ina E. Holroyd, Kansas State College, Manhattan, Kansas
Soap Films—Charles A. Hutchinson, University of Colorado, Boulder, Colorado
Discussion

Tuesday, June 30, 1942

12:15 P.M. Discussion, Luncheon, Olin Hotel, Price: $1.25

It is very important that luncheon reservations be made in advance. Reservations, with an indication of first and second choice of tables, should be sent to Miss Grace Kenehan, Morey Junior High School, Denver, Colorado.

The luncheon will be a very informal occasion. A host and a discussion leader have been designated for each table but no topics have been selected for discussion.

2:00 P.M. Section I. Arithmetic in These Times

Presiding: R. L. Morton, First Vice-President
Ohio University, Athens, Ohio
Do Our Elementary Arithmetic Programs Show Real Concern for Pupil Maturity and Need?—Mrs. Zella K. Flores, Elementary Supervisor, Lewistown, Montana
A Comparative Study of Incidental Number Situations in Grades One and Two—Florence Reid, College of Education, The University of Wyoming, Laramie, Wyoming.
Arithmetic in a Program of General Education—R. L. Morton, Ohio University, Athens, Ohio
Discussion

2:00 P.M. Section II. Junior High School Mathematics in These Times

Presiding: Miss Edith Woolsey, Minneapolis
The Need of Mathematics in the Junior High School—Nona E. Mahoney, Junior High School, Denver, Colorado
Junior High School Pupils' Reactions to Percentage Problems—Glennie Bacon, College of Education, The University of Wyoming, Laramie, Wyoming

* Visitors welcome. Members and visitors please register. There is no charge.

Tailoring Mathematics To Fit the Community—Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin
Discussion

2:00 P.M. Section III. Senior High School Mathematics in These Times

Presiding: O. H. Rechard, University of Wyoming, Laramie
The Double Track Program in High School Mathematics—Harl R. Douglass, Director, College of Education, University of Colorado, Boulder, Colorado
Mathematics As a Useful Language—J. C. Stearns, Ryerson Physical Laboratory, The University of Chicago, Chicago, Illinois
The Pedagogical Significance of the Nature of Geometry—A. E. Mallory, Colorado State College of Education, Greeley, Colorado
Discussion

4:00 P.M. Demonstration of Mathematical Films

Presiding: Mary A. Potter, Racine, Wis.

There will be an elaborate exhibit of mathematics showing "Mathematics in the World of Tomorrow" in the Boy's Gymnasium of East High School set up by the Euclidean Club.

Committees

This program was prepared under the direction of the First Vice-President, R. L. Morton, Ohio University, Athens, Ohio. Valuable assistance was received from the following chairmen of the sectional program committees:
Senior High School: Martha Hildebrandt, Proviso Township High School, Maywood, Illinois
Junior High School: Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota
Arithmetic: Ben A. Sueltz, State Normal and Training School, Cortland, New York
Multi-sensory Aids: E. H. C. Hildebrandt, State Teachers College, Upper Montclair, New Jersey

Local Arrangements Committee
Wendell I. Wolf, Morey Junior High School, Denver, Colorado, Chairman

Luncheon Committee
Grace Kenehan, Morey Junior High School, Denver, Colorado
A. E. Mallory, Colorado State College of Education, Greeley, Colorado

Outing Committee
Ernest R. Bails, Morey Junior High School, Denver, Colorado

Publicity Committee
Valworth R. Plumb, Gove Junior High School, Denver, Colorado
Courtland Washburn, Erie High School, Erie, Colorado

Hospitality Committee
Alfhild M. Alenius, South High School, Denver, Colorado

Exhibits Committee (?)
Harry W. Charlesworth, East High School, Denver, Colorado

EDITORIALS

How Important Is Mathematics?

THERE HAS been a great deal of discussion lately about the importance of mathematics in the secondary schools of this country. If you have never failed the subject or have never had any unfortunate experience with arithmetic, algebra, geometry, or any other branch of mathematics, you may be inclined to agree with those who say that mathematics is an important subject which should be made available (though not necessarily required) throughout the secondary school. If, however, because of failure or poor teaching, or mere lack of interest, you have not seen any of the beauties of mathematics, you will probably agree with those who say that mathematics beyond the bare essentials of arithmetic should not be included, much less required, in the secondary school. There may be a middle group who, neither hating nor liking the subject, may take sides with either one of the groups mentioned above.

It certainly would be interesting, and perhaps enlightening, if some person or group of persons were to find out how many of all those who advocate the curtailment of mathematics, if not its actual elimination from the schools, take that stand because of some unhappy experience with the subject one way or another. Such studies as have been made to ascertain why pupils like or dislike certain subjects have usually shown that in many cases pupils like or dislike a subject (and mathematics is no exception) because they liked or disliked the teacher in question. For that matter it is doubtful if mathematics is any worse taught than other subjects, but the best is none too good. When one considers that the high school population has increased from about 500,000 in 1900 to around 7,000,000 at the present time he can easily see why the problem has become acute.

Again, when some critics of mathematics (particularly the general educators) attack a particular branch of mathematics like algebra, for example, they no doubt have in mind an outmoded type of algebra with which they struggled with little or no success and which we would all agree no longer satisfies present needs. In view of the emergency and the intelligent demands of peace times, a new deal in mathematics as well as in the other great fields of knowledge is necessary.

It is clear to many of us that much of the obsolete material all along the line must be eliminated and its place taken by new material suited to present needs and the entire course reorganized along general mathematics lines for teaching in the schools. It is only in this way that the time wasted by unnecessary reviews of material from one year to the next can be avoided when the subjects are taught in separate compartments.

After several years of enforced retrenchment mathematics is now coming to the front again as a fundamental part of secondary education because of its importance in the war effort. It is too bad that more people did not realize this sooner. Both the Army and Navy are demanding greater attention to the study of mathematics in the secondary schools and are pointing out the deficiencies of our recent program. However, the Army and Navy are not asking teachers in the secondary schools to teach the more technical matters pertaining to war, but only the fundamental parts that should be included in any modern course. These branches of the service and the Air Corps as well will train recruits in technical matters when they

? not less than

come to them. The following quotations will be of interest here:

> The high school is not faced with a depletion of male students to such an extent as is the college. You will not be required to dress up a lot of subjects in uniforms and parade them through catalogues as decoys to get students. Your algebra will not have to be starred as defense algebra, nor will your physics have to be called military physics. Yet you are desirous of doing your part and I can assure you that the War Department considers it most important that the high school student take advantage of every minute of education available to him now and in the future, eventually including a college course. If there must be a fanfare and military glamour present to attract, then let's have it; but the War Department hopes that the normal educational activities of the high schools will be interrupted as little as possible. It is still believed that a healthy training of mind and body is the best contribution the schools can make. The War Department cannot ignore the value of other subjects in Pre-Induction Training, but there are many subjects, dressed in khaki sometimes, that will have little or no application in the service, and we would like to offer a request that you not temporize with the situation. That you not stampede in all directions seeking some magic course because you will very likely find you have the essentials already in your curriculum.
>
> The War Department believes that purely training can best be done by the Army itself after the solider is enrolled. The Army has excellent schools teaching all the special subjects, and its methods of teaching the most elementary duties are up-to-date. All men accepted into the Army have had some Pre-Induction Training and their progress in their new profession will reflect that training. Army training for the vast majority of our soliders is simple and highly repetitious. The technique of most of the weapons is learned by rote. When we enter the realm of tactics, which is the essence of the art of war, we realize the need for imagination and ingenuity in the private soldier and especially in the leader.[1]

[1] From a speech by Colonel B. W. Venable, War Department, Washington, D. C. which was delivered before the Annual Convention American Association of School Administrators, on February 24, 1942 in San Francisco, California.

> So the Navy asks that you encourage every male student under your supervision, who has the slightest aptitude for such subjects, to take an absolute minimum of two and one-half years of mathematics. Teach him algebra through quadratics, plane and solid geometry, and trigonometry. Ground him well in chemistry and physics. Give him practice in machine shop work, foundry work, metalworking, woodworkings, and internal combustion engineering.
>
> Yes, mathematics—and science and manual subjects as well—are acutely necessary in the Navy. But they will be equally necessary in the peace that follows. The forces of destruction that have been unleashed upon the world have been planned by the most scientific minds in the world. The minds which rebuild after this destruction must be equally scientific and even more ingenious. It will take educated and trained men to heal civilization's wounds.
>
> Wherever it is feasible, you can help the Navy by instituting certain temporary elective courses in addition to the regular curriculum. A short, practical course in instruction in the International Morse Code is among those needed. Other valuable subjects would be elementary principles of radio communication, principles of electricity and of navigation, and naval history.
>
> You can help a great deal by making it clear to the students how these subjects will help them serve their country when their time comes. You need make no secret of the fact that a knowledge of these subjects is apt to speed up advancement in the Service. You might do this by a short talk in assemblies at the beginning of each semester, by personal talks with those undecided on their schedules, by bulletins or even posters on your bulletin boards, or by mimeographed general announcements. You probably have all seen the folder on this subject prepared by the Navy. It enumerates the specific educational prerequisites for each specialized job in the Navy, together with an outline of the training the Navy itself offers. See that every male student sees one of these folders. If any additional ones are desired, they may be obtained by writing the Training Division, Bureau of Navigation, Navy Department, Washington, D. C.[2]

[2] From a speech by Lieutenant Commander Paul C. Smith, United States Navy, Washington, D. C. delivered before the Annual Convention American Association of School Administrators, February 24, 1942 in San Francisco, California.

Supplementary Teaching Materials Relating to Aviation

A MATHEMATICS subcommittee of a large general committee on aeronautical education of the Civil Aeronautical Association headed by Dr. N. L. Engelhardt of Teachers College, Columbia University has been given an assignment that will be of interest to teachers of secondary mathematics. The Committee has already outlined two main tasks:

The first job of the committee is to glean from the literature on aeronautics such mathematics problems and computa-

tions as can be readily presented to and understood by the teachers and pupils in their present mathematics classes to classify these according to grade level and topic, to compile them in lists, perhaps in separate pamphlets by topics or grade level, these to be placed in the hands of teachers and pupils as promptly as possible as source material from which teachers may draw problems and practice material to supplement the textbook or syllabus.

For example, in studying *area* the pupils have to find the area of a floor or carpet. They might also be given problems in finding the approximate area of an airplane wing. Again supplementing the making of graphs to show comparative data and the production of wheat, corn and so on, they might draw graphs showing the growth in production of airplanes, number of pilots, and so on.

The purpose of this first job, as indicated above, is to furnish teachers who know little or no aeronautics with materials applicable to the subject which they can introduce immediately into the classroom, (1) to give the pupils some of the knowledge, ideas, and vocabulary relating to aeronautics for which they feel a present need, (2) in order to provide greater variety of applications of the mathematics being taught, (3) to give those pupils who will later take more intensive courses in aeronautics a beginning in the kinds of computations which will be required of them in such courses and, (4) to make children even more air-minded than they are now.

The second job of the committee is to cooperate with other subcommittees in science, industrial arts, and the like in providing the schools with large units of instruction in aeronautics which can be introduced into the course of study and taught more or less as separate subjects.

These units of instruction may range all the way from a unit occupying one or two class periods up to a unit occupying a semester.

There are several distinct advantages in the use of units of instruction as enumerated above.

The committee expects to work in close cooperation with Dr. Paul Mort's committee on the preparation of materials for the ATCA in order that the projects and units of instruction prepared by our committee will tie in to those of Mort's committee. There should be continuity between the work of pupils below ATCA age and those of the ATCA.

The first job, namely, that of discovering of applications of mathematics suitable to be introduced directly into the classroom would best be done by persons with teaching experience since they would know not only what the children could understand in the way of basic concepts underlying these computations, but also how to grade and classify these problems and exercises for presentation to their pupils.

On the other hand, a considerable portion of the work of formulating units of instruction such as the construction of airplane models, wing sections, etc. would have to be done jointly by persons of teaching experience and persons of considerable aeronautical knowledge and experience. For example, a project in the determination of the lift and drag of an airplane model calls for a fairly specialized knowledge of the manner in which such an experiment is conducted and how it could be set up in a manner suitable for pupils. Similarly it would be presumptuous of course for a teacher or other person to formulate a project in construction of a glider who would not have had previous experience in the construction of gliders.

We believe that in order to inaugurate this program a number of teachers colleges in this country should offer courses to teachers during the summer on the teaching of these units of instruction and, if possible, these courses should be subsidized, at least in part, by the Civil Aeronautics Authority.

The third job of the Committee is to

make recommendations concerning: (1) omissions from, and (2) additions to the present mathematics curriculum.

Watch THE MATHEMATICS TEACHER for further announcements about the work of this and other similar committees.

A printed report of the work of this Mathematics Committee will be ready for free distribution around July 1, 1942. Copies of the report may be obtained by writing directly to Dr. Engelhardt.

W.D.R.

A Message from Past President Potter

LIKE GOVERNMENT, education is continually facing a fresh crisis. In this last crisis created by the Pearl Harbor attack, mathematics is assigned an unaccustomed role when leaders in the armed forces and industry replace the mathematics teacher in urging mathematical training for our young people. These laymen are publicizing the fact that a knowledge of mathematics is essential for successful warfare; it remains for us to equip our students with a practical working knowledge of the subject. This is our opportunity as well as our responsibility, an opportunity in which you, the Council, must assume leadership to assure a new, sane, and far seeing program of education in mathematics.

Especial thanks should be given to the wise and devoted Board of Directors and past presidents who have planned the activities of the Council, to the chairmen and members of the standing committees who have contributed in carrying out these plans, to those members who have made possible the conventions by efficient committee work and inspiring addresses, to the state representatives who are on the firing line, and to the "unknown soldier" who presents mathematics to the youth of America. With this support and with the competent leadership of your new president, the Council can meet the challenge of 1942.

MARY A POTTER

IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

The American Mathematical Monthly

February, 1942, vol. 49, no. 2.

1. Denbow, Carl, "Means and Ends in Mathematics," pp. 105–106.
2. Kac, Mark, and Randolph, J. F., "Differentials," pp. 110–112.
3. Price, Irene, "I Doubt It—a Mathematical Card Game," p. 117.

Bulletin of the Kansas Association of Teachers of Mathematics

February, 1942, vol. 16, no. 3.

1. Ulmer, Gilbert, "To the Memory of our Associate Editor, Friend and Leader, U. G. Mitchell," p. 35.
2. Read, Lyle, "A Unit on Assumptions in Plane Geometry," pp. 37–39.
3. Mossman, Thirza A., "Discoveries in Mathematics That Have Changed the Course of Civilization," pp. 39–40.
4. Potter, Mary, "In Defense of Donald the Dull," pp. 40–45.
5. Hartley, Willa, "Importance of Drill and How to Secure Needed Remedial Work," pp. 45–46.
6. Sherman, Della, "What to Omit in Grade Six and How to Base the Teaching of Percentage on Known Material in Grade Seven," p. 48.

National Mathematics Magazine

February, 1942, vol. 16, no. 5.

1. Dorwart, Harold L., "Beyond Quadratics," pp. 231–237.
2. Goodman, Adolph, "On Integers of The Form $\frac{2^{p-1}-1}{p}$," pp. 238–239.
3. Dunnington, G. Waldo, "U. G. Mitchell, 1872–1942," pp. 240–242.
4. Sleight, E. R., "Early English Arithmetics," pp. 243–251.
5. Erskine, William H., "The Use of Index Numbers in Evaluation," pp. 252–258.

School Science and Mathematics

March, 1942, vol. 42, no. 3.

1. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 239–242.
2. Sands, Lester B., "A Self-Rating Check Sheet for Progressive Practice in Elementary Mathematics and Science," pp. 263–267.
3. Gordon, David X., "Clarifying Arithmetic through Algebra," pp. 286–289.

Miscellaneous

1. Blackwell, A. M., "Comparative Investigation into the Factors Involved in Mathematical Ability of Boys and Girls," *British Journal of Educational Psychology*, 10: 143–153, 212–222, June-November, 1940.
2. Boyer, L. E., "Appreciations in Arithmetic," *Educational Method*, 21: 199–202, January, 1942.
3. Burington, R. S., "Training of Mathematicians," *Journal of Engineering Education*, 32: 346–352, December, 1941.
4. Held, O. C., "College Mathematics Placement Test," *Journal of Higher Education*, 13: 39–40, January, 1942.
5. Jones, E. M. and Powers, M., "Embellishing Euclid," *Educational Method*, 21: 97–98, November, 1941.
6. Pounder, I. R., "Application of Geometry in Air Navigation by Dead Reckoning," *School*, (Secondary Edition), 30: 211–215, November, 1941.
7. Thomas, C. S., "Counsel for the Defense of Mathematics," *Ohio Schools*, 20: 22, January, 1942.

NEWS NOTES

The joint luncheon and panels of the Mathematics Chairmen's Association and the Association of Teachers of Mathematics of New York City were held at the Hotel Astor on March 21, 1942; panel discussions: 10:30 A.M. Luncheon: 12:30 P.M. The program follows:

Panel I: Preparation in Mathematics for Participation in the War EZort
Chairman: Mr. Max Peters, Lafayette High School

(a) "A Survey of Defense Activities in New York City Mathematics Departments." Mr. Joseph B. Orleans, Chairman of the Standing Committee for Mathematics.

(b) "Mathematical Instruments Used in Defense Activities." Dr. Fred L. Bedford, New Jersey State Teachers College.

(c) "Ballistics." Mr. Gordon R. Mirick, Principal, Lincoln School.

Panel II: The New Syllabus for the 7th, 8th and 9th Years
Chairman: Miss Grace Carlin, Jr. H. S. 128, Brooklyn

(a) "Historical Background of the Syllabus." Mr. Albert E. King, Principal, Jr. H. S. 259, Brooklyn.

(b) "Unifying the Mathematics in the New Syllabus." Mrs. Lorraine W. Addelston, Chairman of the Math. Dept., Jr. H. S. 159, Man.

(c) "Teaching Indirect Measurement in the Ninth Year." Mr. Saul Landau, Jr. H. S. 139, Man.

(d) "Problems Connected with the Supervision of the New Syllabus." Dr. Rufus M. Hartill, Assistant Superintendent.

(e) "The New Philosophy of Education in Relation to the New Mathematics Syllabus." Dr. Jacob Theobald, Assistant Superintendent.

Panel III: Tenth Year Mathematics Articulation with the New Syllabus for the 7th, 8th and 9th Years
Chairman: Miss Ethel R. Marks, Evander Childs High School

(a) "What Are the Objectives of the 10th Year?" Dr. Harry Eisner, Chairman of the Math. Dept., Manual Training High School.

(b) "What Algebra Should Be Included?" Miss Theresa Molloy, Chairman of the Math. Dept., William Cullen Bryant H. S.

(c) "What Are the Integration Possibilities?" Miss Elizabeth Sibley, Andrew Jackson H. S.

(d) "How Can We Utilize the Background in Experimental Geometry to Develop the Nature of Proof and the Concept of a Postulational System?" Mr. Harry Sitomer, Chairman of the Math. Dept., New Utrecht H. S.

(e) "What Changes Should Occur in Regents Examinations?" Mr. F. Eugene Seymour, Supervisor of Mathematics, Albany, New York.

Panel IV: Methods of Teaching Selected Topics
Chairman: Mr. Barnett Rich, High School of Music and Art

(a) "Enrichment of Trigonometry through Integrated Mathematics." Miss Agnes Morley, Andrew Jackson High School.

(b) "Definitions in Geometry—Their Uses and Abuses." Mr. Samuel Welkowitz, Chairman of the Math. Dept., Franklin K. Lane High School.

(c) "Regular Polygons." Mr. Herman Karnow, Midwood High School.

(d) "Mathematics in Bombing and Aerial Gunnery." Mr. Henry Mayer, James Madison High School.

(e) "A Project Scrap Book—Mathematics and Life." Mr. Leon J. Blay, Midwood High School.

Panel V: Mathematics in the Vocational High Schools
Chairman: Mr. Samuel Altwerger, Bronx Vocational High School

(a) "The Role of Mathematics in the Vocational High School." Mr. Morris Shapiro, Assistant to the Director of Vocational Schools.

(b) "General Mathematics for Girls in the Vocational High Schools." Mrs. Pearl G. Bennett, Teacher-in-Charge, Brooklyn H. S. for Women's Garment Trades.

(c) "General Mathematics for Boys in the Vocational High Schools." Mr. Joseph Tieger, Chairman of the Math. Dept., Brooklyn H. S. for Specialty Trades.

(d) "Related Mathematics in the Vocational High Schools." Dr. David E. Brownman, Teacher-in-Charge, Murray Hill H. S. for Building and Metal Trades.

Luncheon Program: "The Maximum Scientific Effort in Total War," an address by Porfessor Marston Morse.

The Committee: Samuel Altwerger, Grace Carlin, Eugenie C. Hausle, Morris Hertzig, and George J. Ross.

On the Dias: Mr. Albert E. King, Principal of Jr. H. S. 259; Mr. Morris Shapiro, Ass't. to the Director of Vocational High Schools; Prof. T. Freeman Cope, Queens College; Mr. Gordon R. Mirick, Teachers College; Prof. William D. Reeve, Teachers College; Mr. Joseph Jablonower, Board of Examiners; Mr. Fred Schoenberg, High School Division, Board of Education; Mr. Herman Wright, Assistant Superintendent; Prof. Marston Morse, Institute of Advanced Study; Prof. William S. Schlauch, New York University; Dr. Eugenie C. Hausle, President Mathematics Chairmen's Association; Dr. Nathan Lazar, Pres. Ass'n. of Teachers of Mathematics of N. Y. C.; Dr. Rufus M. Hartill, Assistant Superintendent; Dr. John L. Tildsley, Associate Sup't. (Retired); Prof. Edward Kasner, Columbia University; Prof. Richard Courant, New York University; Dr. Jacob Theobald, Assistant Superintendent; Mr. Joseph B. Orleans, Chairman of Standing Comm.; Mr. F. Eugene Seymour, Supervisor of Mathematics, Albany; Dr. Fred L. Bedford, New Jersey State Teachers College.

Mr. Fred Schoenberg, Toastmaster
Dr. Eugenie C. Hausle, Presiding Officer

Professor W. D. Reeve of Teachers College, Columbia University, was the guest speaker at the Mathematics Section of Schoolmen's Week at the University of Pennsylvania on March 19, 1942.

News from the Winthrop chapter of the National Council of Teachers of Mathematics.

The Winthrop chapter of the National Council of Teachers of Mathematics has been fortunate this year in having many excellent discussions led by members of the club. At the beginning of the year, the program committee decided that two papers should be given at each meeting: one of these papers to be of a pedagogical typeand the other some topic from advanced mathematics.

The following are among the pedagogical discussions enjoyed by the club: "The place of Mathematics in Secondary Education," "The Lesson Non-Euclidean Geometry Can Teach," "Extracurricular Mathematical Activities in Secondary Schools," "The Mathematical Christmas Party," and "Recent Trends in Arithmetic."

In advanced mathematics the following topics have been discussed: "The Problem of the Duplication of the Cube—Its Solution by the Cissoid of Diocles," "The Trisection of an Angle—Solution by the Conchoid of Nicomedes," "The Fundamental Theorem of Rational Arithmetic," "The Geometry of Euclid: Visual Experience or Exact Science?" a discussion on "Prime Numbers," and Dr. Ruth Stokes, whom the club is very fortunate to have as a member, reported on the meetings in Baton Rouge and Atlantic City in a special discussion of "Mathematics in Defense."

This year the club has had as its faculty advisor Dr. Norman Royall, who has been helpful in promoting interest in the club and adding to the instructional value received therefrom.

The officers of the club are: President, Helen Hanna; Vice-president, Sarah Burgess; Secretary and treasurer, Betty Brown; Reporter, Alice Martin.

Assign Second Set of Planes for Model Building Program

With American school youth well started toward the goal of making 500,000 scale models of United Nations and Axis warplanes, the U. S. Office of Education, in conjunction with the Navy Bureau of Aeronautics, today released names and designations of the second set of 20 types of planes to be modeled.

Public- and private-school students are making 10,000 models of each of 50 types of aircraft for use in range estimation, gunnery, and identification training for personnel of the Navy, Army, and civilian defense forces.

Thousands of Model Aircraft Project directors, representing school districts in every section of the country, received plans and specifications for the first set of 20 types on February 23, and the 200,000 completed models in this set are expected to begin arriving at receiving centers late this month.

Plans for the second set of 20 planes named today will be mailed to local project directors about March 20 and the last 10 types about the middle of April, the U. S. Office of Education announced.

The 20 types of planes assigned today represent six nations, three Allied and three Axis. Russian and Italian planes are introduced for the first time, along with additional models from the four nations—United States, Britain, Germany, and Japan—represented in the first assignment.

These are the 20 announced today:

U. S. Navy—Martin PBM (Mariner), Brewster SB2A-1 (Buccaneer), Curtiss SOC-3, scout observation, and Grumman J2F, amphibian.

U. S. Army—Lockheed P-38 (Lightning), Martin B-26 (Maryland), Consolidated B-24 (Liberator), Republic P-43 (Lancer).

U. S. Commercial—Lockheed Lodestar.

British—Hawker Hurricane, Bristol Blenheim.

Russian—I-16, fighter.

German—Messerschmitt Me-110, Junkers Ju-87B, Junkers Ju-88A-1, Dornier Do-18.

Japanese—Mitsubishi fighter 96, Kawasaki 97, Nakajima 95.

Italian—Savoiia-Marchetti SM-82 (Cangaru).

J. C. Wright, Assistant U. S. Commissioner of Education, who is directing the project for the U. S. Office of Education, reported that "we have had requests from representatives of newspapers, magazines, model aircraft clubs, commercial airlines, and from individuals for sets of the plans. In every case we refer them to their State or their local model aircraft project director.

"The local director may permit persons to examine the plans in his office, or he may, at his discretion, loan the plans temporarily to persons who want to reproduce them."

Dr. Wright re-emphasized, however, that the schools bear responsibility for building the half-million accurate-scale model planes for the use of our armed forces, and that nothing must deflect the schools from their goal.

"We could probably get ten times the number of planes the Navy has asked for by authorizing patriotic model plane clubs and similar organizations to join in this job," said Dr. Wright. "But the problem was to get the proper sets of models made exactly to scale, to get them quickly and without confusion at the receiving end.

"Individuals, whether in school or not, may submit their models to local project directors, and if the models are approved by the inspection committee they may form part of the local quota," Dr. Wright said. "But we have released shipping instructions only to project directors so that receiving centers will not be burdened with an overflow of models."

Both the U. S. Office of Education and the Bureau of Aeronautics stressed that the Navy does not want and cannot use planes constructed according to plans other than those provided by the Navy through the U. S. Office of Education. Only models within the local quota, passed by official inspection committees, and shipped by the local project director will be accepted by the Navy.

In instructions to local model aircraft project directors the Navy cautioned that sets of models be packed carefully because original cartons may be reshipped from receiving centers to distant points.

The First Annual Meeting of the Metropolitan New York Section of the Mathematical Association of America was held at Hunter College on Saturday, April 18, 1942.

Program

Morning Session, 9:30 A.M.

Chairman, Professor Frederic H. Miller, Cooper Union.

Address of Welcome—President George N. Shuster, Hunter College.

"An Application of Matrix Theory to Cryptography," Professor L. S. Hill, Hunter College.

"The Teaching of Mathematics at the Defense Training Institute," Mr. Charles H. Lehmann, Cooper Union.

"On the Principles of Statistical Inference," Dr. Abraham Wald, Columbia University and Queens College.

"Mathematical Training for Aeronautical Engineers," Dr. Newman A. Hall, Vought Sikorsky Aircraft.

Luncheon, 12:15, Hunter College Cafeteria. The executive committee will meet at luncheon.

Afternoon Session, 1:30 P.M.

General Chairman, Professor T. Freeman Cope, Queens Collge.

Election of Officers.

Symposium on Integrated Mathematics in High School.

Chairman, Dr. John A. Swenson, Andrew Jackson High School.

"The Significance of Δx in Secondary Mathematics," Miss Agnes Morley, Andrew Jackson High School.

"Integrated Mathematics with Special Application to the Tenth Year (Geometry)," Mr. Harry Sitomer, New Utrecht High School, Brooklyn.

"Spatial and Probable Relationships in Secondary Mathematics," Dr. Edna Kramer, Thomas Jefferson High School, Brooklyn.

"Integrated Mathematics in Catholic High Schools," Brother Anselm, St. Joseph's Normal Institute, Barrytown, N. Y.

Officers of the Metropolitan N. Y. Section: T. Freeman Cope, Queens College, Chairman; John A. Swenson, Andrew Jackson High School, Vice-chairman; Howard E. Wahlert, New York University, Secretary; Frederic H. Miller, Cooper Union, Treasurer.

The Heads of State Departments of Education and Administrators of Elementary and Secondary Schools

The wholehearted and prompt response of school officials to the Navy Department's suggested Secondary School Program has been most gratifying. The Navy again urges all shools to

strengthen their courses in mathematics and the sciences.

Cooperation in this Program can best be effected by local school authorities through regular school channels, such as State Departments of Education, State Superintendents of Education, the United States Office of Education, and similar sources. The Navy Department will therefore clear all information throuth the Educational Department of each state. This will avoid the possibility of confused situations and prevent duplication of efforts.

The following is quoted from the Mathematical Education for Defense, report of June-July, 1941: "An emergency justifies any remedial action, but our efforts should be directed toward making it unnecessary to use hazy emergency shortcuts to mathematical procedures. With out widespread democratic system of secondary and collegiate education, our nation is justified in demanding that we should always have on hand a relative surplus of people with mathematical training through substantial secondary mathematics and also a surplus with elementary college training in the subject."

The cooperation of every school administrator and teacher in the Navy's suggested Program in the schools through their state Education Department is deeply appreciated.

RANDALL JACOBS,
Rear Admiral, U. S. Navy,
Chief of Bureau of Navigation

F. U. LAKE
Captain, U.S.N. (ret.),
Director of Training Division
By direction.

The Association of Mathematics Teachers of New Jersey helds its Seventy-fourth Regular Meeting at State Teachers College, Newark, New Jersey, Saturday, March 7, 1942.

General Theme: Mathematics and Social Effectiveness

A. Section Meetings 10:15–11:45 A.M.

I. College Section. Dr. Howard F. Fehr, presiding.

Topic: The Content of a One-year Course in College Mathematics.

Speakers: Professor Albert E. Meder, Jr., New Jersey College for Women, New Brunswick, N. J. On Pure Mathematics. Professor Martin A. Nordgaard, Upsala College, East Orange, N. J. On Cultural and Historical Content. Dr. Virgil S. Mallory, State Teachers College, Montclair, N. J. For Teacher Colleges. Discussion from the floor.

II. Senior High School Section. Mrs. Florence Gorgens, presiding.

Topic: Measurement in Industry and in the Military Services.

Speakers: Mr. Q. H. Welsh, Courtesy of International Business Machine Corporation, Endicott, N. Y. Dr. Carl N. Shuster, State Teachers College, Trenton, N. J. Discussion from the floor.

III. Junior High School Section. Hubert B. Risinger, presiding.

Topic: Promising Trends and Meritorious Developments.

Part I. Dynamic Curricula.

Chairman: Mary C. Rogers, Roosevelt Junior High School, Westfield, New Jersey. Contributors: Dr. E. H. C. Hildebrandt, State Teachers College, Montclair, New Jersey, Eva Trumpbour, Davey Junior High School, East Orange, New Jersey.

Part II. Is Drill Obsolete?

Chairman: Edwin K. Cunliffe, Defense Engineering Division, Rutgers University, New Brunswick, New Jersey.

Contributors: Margaret Cleary, Junior High School, No. 3, Trenton, New Jersey, Margaret Aeschbach, Junior High School, East Orange, New Jersey, Ester F. Maltenfort, Memorial Junior High School, Passaic, New Jersey, Addie L. Taylor, Junior High School, Long Branch, New Jersey. Discussion from the floor. Summary: Dr. Foster E. Grossnickle, State Teachers College, Jersey City, New Jersey.

B. Motion Pictures, 12:00–2:00 P.M.

12:00—World's Largest Telescope.

12:15—Stereoscopic Mapping from the Air.

12:40—The Origin of Mathematics.

C. Afternoon Session, 2:00 P.M. Dr. Howard F. Fehr, presiding.

I. Announcements.

II. Greetings—Dr. Roy L. Shafer, State Teachers College, Newark, N. J.

III. Address—Mathematics—a Force in Social Effectiveness. Dr. William D. Reeve, Professor of Mathematics, Teachers College, Columbia University.

An institute for teachers of secondary mathematics will be held at Duke University in Durham, North Carolina from June 16 to 26, 1942 under the direction of Professor W. W. Rankin with an executive committee composed of: Professor Helen Barton, Woman's College U. N. C.; Professor E. F. Canaday, Meredith College; Miss Bonnie Cone, Charlotte High School; Miss Laura Efird, Raleigh High School; Professor H. F. Munch, University of North Carolina; Professor Ruth Stokes, Winthrop College; and Professor W. W. Rankin.

Registration will take place from 9:00 A.M. to 3:00 P.M. on June 16 in the Social Room of the Union Building on the West Campus. There will be a fee of one dollar for registration. This

includes a special class in "Mathematics for Defense" consisting of ten non-credit lessons. This class will meet daily at 8:00–9:30 in Room 210 School of Religion beginning on June 16.

The work in this class will emphasize the applications of secondary mathematics to air and sea navigation, sound-ranging, map-making, artillery firing, computing devices, use of tables, instruments, and so on. The class will be taught by both Naval and Civilian instructors.

The following persons will participate in the conference: Professor W. W. Rankin, Dr. J. Henry Highsmith, Mr. William Betz, Dr. John W. Carr, Dr. E. H. Garringer, Professor W. A. Brownell, Professor E. Cameron, Miss Laura Effrd, Mr. K. B. Curtis, Professor A. M. Proctor, Miss Lenore John, Mrs. Stack, Mr. John W. Moore, Miss Carrie Wilson, Mr. Randolph Benton, Mr. H. A. Perry, Professor H. F. Munch, Miss Olive Smith, Mr. Ray Armstrong, Miss Anderson, Mr. M. E. Yount, Professor Helen Barton, Professor Mumford, Professor E. T. Browne, Professor K. B. Patterson, Professor W. W. Elliott, Professor E. F. Canaday, Professor J. J. Gergen, Dr. R. L. Flowers, Miss Eura Strother, Professor Ruth Stokes, Professor J. M. Thomas, Lt. Com. W. C. Cron, Professor F. G. Dresel, Dean A. W. Hobs, Professor E. L. Mackie, Miss Bonnie Cone, Professor Douglas E. Scates, Professor W. J. Seeley, Professor Myer, Miss Rosalie Elliott, Professor Olive M. Jones, and Professor B. G. Childs.

Dr. Nathan Lazar of the Bronx High Sohool of Science addressed the Mathematics Section of the Rockland County Association of Teachers at Spring Valley, New York on Friday, March 27, 1942. The topic was "What kind of Mathematics Should be Taught During the Present Emergency." Mr. Vincent Festa of Congers, New York presided.

Recent Meetings of the Men's Mathematics Club of Chicago and Metropolitan Area for 1941–42 were held as follows:

December 19

"Algebra for Teachers," by Professor Frank E. Wood of Northwestern University.

January 16

"Mathematical and Physical Principles of Modern Weather Forecasting" by Victor Starr, University of Chicago.

February 13, Ladies Night

"A Million Ways to Solve Equations" by Dr. Lester R. Ford.

March 20

"Navy Mathematics" by Commander Cobb, Professor of Naval Science and Tactics at Northwestern University.

April 17

"Streamling Solid Geometry for Defense" by Hans Gutekunst of Batavia High School.

"A New Visual Aid for Teaching Mathematics" by Edwin W. Schrieber of State Teachers College, Macomb, Illinois.

The last meeting of the year will be held on May 15.

We regret to announce the passing of two of Chicago's finest teachers of mathematics, Mr. M. J. Newell of Evenston Township High School, and Mr. William H. Clark of the Gage Park High School. Both of these men were members of long standing and past presidents of the Men's Mathematics Club.

R. E. Anspaugh
Chairman, Program Committee
S. F. Bibb
Secretary

The Twenty-Eighth Annual Meeting of the Kansas Section of the Mathematical Association of America and the Thirty-Eighth Annual Meeting of the Kansas Association of Teachers of Mathematics was held at Fort Hays Kansas State College on March 27–28, 1942. The program follows:

FRIDAY, MARCH 27

Evening Session

8:00 p.m.—Science Hall, Room 209
C. F. Lewis, Presiding

I. Continued Fractions of Quaternions.
Earl G. Swafford
Fort Hays Kansas State College

II. A System of Linear Differential Equations with a Regular Singular Point.
Frank Faulkner
Kansas State College, Manhattan

III. Some Properties of Sine Z as the Inverse of an Integral.
B. H. Buikstra
Kansas State College, Manhattan

IV. Subject to be announced.
G. Baley Price
University of Kansas

"Get Acquainted Hour"

SATURDAY, MARCH 28

Morning Session

9:30 a.m.—Science Hall, Room 209
C. F. Lewis, *Presiding*

I. Mathematics in The Armed Forces.
Daniel T. Sigley
Kansas State College, Manhattan

II. Some Circles Related to a Triangle.
G. W. Smith
University of Kansas

III. Is Mathematics An Exact Science?
C. B. Read
University of Wichita

IV. Report of Committee on Placement Test.
Gilbert Ulmer
University of Kansas

LUNCHEON AND BUSINESS MEETING
12:00 Noon—Cody Commons

AFTERNOON SESSION
K. A. T. M.—2:00 P.M.
Science Hall Room 210
KATHLEEN O'DONNELL, *Presiding*

Teaching Geometry to Develop Clear Thinking.
Gilbert Ulmer
University of Kansas
Discussion

KANSAS SECTION, *M. A. A.* 2:00 P.M.
Science Hall, Room 210
C. F. LEWIS, *Presiding*

Report of Committee on Placement Test.
O. J. Peterson
Kansas State Teachers College, Emporia
Discussion
Business Meeting

Officers Kansas Section M. A. A.

Chairman—C.V. Bertsch, Southwestern College Winfield

Vice Chairman—C. F. Lewis, Kansas State College, Manhattan

Secretary—Lucy T. Dougherty, Kansas City Jr. College, Kansas City, Kansas

Nominating Committee—G. W. Smith, University of Kansas, Lawrence; Anna Marm, Bethany College, Lindsborg; C. B. Read, University of Wichita, Wichita

Officers K. A. T. M.

President—Kathleen O'Donnell, Senior High School, Salina

Vice President—Daniel Pease, Wyandotte H. S., Kansas City, Kansas.

Secretary-Treasurer—Forrest Noll, Lawrence H. S., Lawrence

Editor Bulletin—Ina E. Holroyd, K. S. C., Manhattan

Nominating Committee—Maude Long, Lyons; Letha Schoeni, Oberlin; Paul F. Ottens, Hays; Amanda Neuschwanger, Salina, *Chairman*

The Nineteenth Annual Joint Meeting of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics was held at the Heidelberg Hotel in Jackson, Mississippi on March 6 and 7, 1942. The program follows:

Joint Preliminary Meeting

Silver Room, 2:00 P.M., Friday, March 6

Welcome—Dorothy McCoy, Belhaven College
Announcements and Appointments of Committees

The Louisiana-Mississippi Section of the Mathematical Association of America

Silver room, 2:30 P.M., Friday March 6

B. A. Tucker, Southeastern Louisiana College, Chairman, presiding
W. V. Parker, Louisiana State University, Secretary

1. A solid of Revolution—J. A. Ward, Delta State Teachers College
2. Poristic Polygons—H. E. Buchanan, Tulane University
3. Fire Insurance, A One, A Three, or A Five Year Policy?—I. C. Nichols, Louisiana State University
4. The Problem of Apolloius—Alta H. Samuels, Raymond Junior College
5. The Twelve Squares Ascribed to a Triangle—B. E. Mitchell, Millsaps College
6. A Note on Heaviside Operators—J. F. Thomson, Tulane University
7. Distributions in Stratified Sampling—Paul H. Anderson, Louisiana State University
8. Boundary Values for Probabilities in Problems of Two Variables—C. D. Smith, Mississippi State College

Joint Banquet

Victory Foyer Room, 7:00 P.M., Friday, March 6
Welcome—W. E. Reicken, Millsaps College
Response: Dewey S. Dearman, Mississippi Southern College; H. E. Buchanan, Tulane University
Address: The Next Step Forward, Professor F. L. Wren, George Peabody College for Teachers

Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics

Room 726, 8:00 A.M., Saturday, March 7
Dewey S. Dearman, Chairman, presiding
Jessie May Hoag, Secretary

Is the Statistician a Greasy Grind?—Paul H. Anderson, Louisiana State University

Is Subtraction Over-emphasized?—Norman E. Dodson, Jefferson Military College, Washington, Miss.

The Place of Mathematics In The Junior College—W. H. Bradford, John McNeese Junior College, Lake Charles, La.

Some Aspects of the Mathematics Program in Junior College—C. C. Dearman, Jr., East Central Junior College, Decatur, Miss.

What is the National Council of Teachers of Mathematics?—F. L. Wren, George Peabody College for Teachers

General Mathematics in the High School Curriculum—Donald Turner, Belle Rose High School, Belle Rose, La.

Reflections of a Secondary Mathematical Teacher —Ruth E. Ramsey, Amite High School, Amite, La.

Your Big Opportunity—Mrs. L. C. Christensen, Lake Charles, La.

Some Observations of a Mathematics Teacher in Defense Training—R. E. Horne, Hattiesburg High School, Hattiesburg, Miss.

Mathematics—Our Chief Weapon of Defense—James Barton, Central High School, Jackson, Miss.

Louisiana-Mississippi Section of the Mathematical Association of America

Silver Room, 10:30 A.M., Saturday, March 7

A Contraction Method for Determinant Expansion—F. L. Wren, Business Meeting and Election of Officers

Officers 1941–42
Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics

Dewey, S. Dearman, Mississippi Southern College, *Chairman*

Pearl Spann, Central High School, Jackson, Mississippi, *Vice-Chairman*

Jessie May Hoag, Jennings High School, Jennings, Louisiana, *Secretary*

Houston T. Karnes, Louisiana State University, *Recorder*

Louisiana-Mississippi Section of the Mathematical Association of America

B. A. Tucker, Southeastern Louisiana College, *Chairman*

W. E. Cox, Mississippi State College, *Vice-Chairman*

F. A. Rickey, Louisiana State University, *Vice-Chairman*

W. V. Parker, Louisiana State University, *Secretary*

The Mathematics Section of the Louisiana Teachers Association has elected the following officers:

President: Miss Carolyn Rosenthal, Baton Rouge

Vice President: Miss Irma Smart, New Orleans

Secretary: Miss Melva Le Blance, Kaplan

The new officers of the Mathematics Section of the Maryland State Teachers Association which is affiliated with the National Council of Teachers of Mathematics are as follows:

Chairman, Grover Wm. Norris
Vice-Chairman, Anna Meeks
Secretary, Margaret Bowers
Treasurer, Margaret Heinzerling
Assistant Treasurer, J. Karl Schwartz
State Representative, Agnes Herbert

The Spring Meeting of the Mathematics Section of the Eastern Division of the Colorado Education Association was held at Denver on April 11, 1942. The program follows:

Theme: Mathematics Forges Ahead

1. Mathematics as a Contributing Subject.—Amanda A. Lindsey, Byers Junior High, Denver.
2. Is Mathematics Taught Incidentally?—Isabel M. Lody, Gove Junior High, Denver
3. Mathematics at work in a Modern Setting.—Ruth Irene Hoffman, Skinner Junior High, Denver.
4. Mathematics and Defense.—Speaker from Lowry Field.

The Members of the executive council are:

Wendell Wolf—*President*—2831 West 34th Ave., Denver

A. E. Mallory—*Vice President*—C.S.E.C., Greeley

Valworth Plumb—*Secretary-Treasurer*—4763 Raleigh St., Denver

Term expires Nov. 1942

Florence Barnard, Aurora
Courtland Washburn, Erie
J. C. Fitterer, Golden
Mary Doremus, Denver

Term expires Nov. 1943

Margaret McGinley, Denver
Frances Smith, Sterling
Dwight Gunder, Ft. Collins
Ernest Cruse, Greeley

Term expires Nov. 1944

Margaret Aylard, Denver
Dan Beattie, Ft. Collins
Arthur Lewis, Denver
Isabelle Staub, Denver

The Winter Meeting of the Portland Council of Teachers of Mathematics was held Dec. 6, 1941, at the Heathman Hotel, Portland, Oregon.

At the morning session Lieutenant H. R. Johnson, Classification Officer at the Portland Columbia Airbase explained the use made of

tests in placing men in positions for which they are best qualified.

Miss Lesta Hoel, Supervision of Mathematics in the Portland schools gave a paper on "Fractions."

The officers elected for 1942 are:

President: Mr. Robert Main, Washington High School. (Home) 3736 N.E. 80th St., Portland, Oregon.

Vice-President: Miss Katherine Piggott, Jefferson H. S. (Home) 1409 N.E. Hancock St., Portland, Oregon.

Secretary-Treasurer: Mr. A. B. Carter, Grant H. S. (Home) 6635 S.E. Yambill St., Portland, Oregon.

Corresponding Secretary: Miss Agnes Beach, Washington H. S. (Home) 2328 S.W. 18th St., Portland, Oregon.

A survey of present uses and future needs of mathematics teachers is being conducted by Henry W. Syer, Instructor in Mathematics, Culver Military Academy, Culver, Indiana. The results of this survey will form part of the 1943 Yearbook of the National Council on Multi-sensory Aids for the Teaching of Mathematics.

The following questions are being asked of mathematics teachers who are known to be interested in teaching films:

1. Do you use motion pictures for the teaching of mathematics in your school?
2. If so, what pictures have you used?
3. Would you care to comment on their use: for introduction, exposition, summary, supplementary interest, or testing? Were they well received and did they achieve their purpose?
4. What other motion pictures suitable for mathematics classes could you add to a general list for the Multi-sensory Aids Committee?
5. Have you made any pictures for your own use?
6. What type of motion picture which is not available do you feel is most needed at the present time for the teaching of mathematics?
7. Do you know of any commercial company interested in producing films for the teaching of mathematics?
8. Would you be interested in cooperating in a practical experiment to judge the value and place of motion pictures in mathematics classes?

Any teachers interested in helping with this survey are invited to correspond with Mr. Syer (address above).

On Friday, April 17, 1942 the Kentucky Council of Mathematics Teachers met in Louisville, Kentucky with Miss Tryphena Howard of Western Kentucky State Teachers College, Bowling Green, officiating. Dr. H. A. Wright of Transylvania University, Lexington, Kentucky, speaking at the luncheon meeting, discussed "Freshmen Failures on College Entrance Examinations." At the afternoon meeting the principal speaker was Dr. Robert C. Yates of Louisiana State University, who addressed the group on "A Mathematics Laboratory." Illustrating his lecture with instruments from his own laboratory, Dr. Yates challenged the teachers to put mathematics into the hands of their students, so that it would be more meaningful to them.

An exhibit of mathematical instruments was provided for display by Dr. Robert C. Yates, Dr. W. L. Moore, University of Louisville, and Miss Edith Wood, Okolona High School, Louisville.

For the year 1942–43 Mr. C. A. Stokes, Male High School, Louisville, was elected president and Mrs. R. L. Queen Jr., secretary-treasurer.

ALBERTA QUEEN, *Secretary*

Copies of the study *The Status of Teachers of Secondary Mathematics in the United States* by Ben A. Sueltz may be had by writing to him at the State Teachers College, Cortland, New York. These will be sent postpaid at cloth \$1.00 or paper 50¢. This is the comparative study that was made in 1932 for the International Commission on the Teaching of Mathematics and is of value chiefly for the data it contains.

From Eunice Lewis, 2203 East 14th St., Tulsa, Okla. comes the following:

"I feel that the TEACHER has improved more this year than any year since 1928 when I first became a member. This year's issues have something real to offer to struggling mathematics teachers. Compare any issue of this year with those of 1928 and years back. I did while browsing through mine recently—it was a real eye opener."